Online Appendix for:
"Belief Distortions, Asset Prices, and Unemployment Fluctuations*"

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March 15, 2026

Contents

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OA Online Appendix: Additional Results

OA.1 Stylized Facts

Summary Statistics Appendix Table OA.1 summarizes the distributions of survey-based and machine learning forecasts for the key components of the variance decomposition. The most notable pattern is the contrast in time-series volatility and cross-sectional dispersion between the two sources of expectations. In the time series, 5-year survey-based discount rate expectations $\mathbb{F}_t[r_{t,t+5}]$ are substantially less volatile than machine forecasts, with standard deviations of 0.037 and 0.118, respectively. In contrast, 5-year survey-based cash flow expectations $\mathbb{F}_t[e_{t,t+5}]$ exhibit much higher volatility than machine forecasts, with standard deviations of 0.299 and 0.058, respectively.

Table OA.1: Summary statistics

	Obs	Mean	St. Dev.	Min	p25	Median	p75	Max
$r_{t,t+5}$	72	0.284	0.283	-0.279	0.131	0.330	0.464	0.789
$\mathbb{F}_t[r_{t,t+5}]$	72	0.226	0.037	0.147	0.195	0.229	0.251	0.327
$\mathbb{E}_t[r_{t,t+5}]$	72	0.287	0.118	0.036	0.209	0.284	0.362	0.572
$e_{t,t+5}$	72	3.739	0.300	2.353	3.741	3.777	3.905	4.288
$\mathbb{F}_t[e_{t,t+5}]$	72	3.908	0.299	3.264	3.768	3.892	4.101	4.423
$\mathbb{E}_t[e_{t,t+5}]$	72	3.801	0.058	3.704	3.763	3.793	3.823	3.936
pe_{t+5}	72	3.553	0.294	3.084	3.332	3.527	3.642	4.594
$\mathbb{F}_t[pe_{t,t+5}]$	72	3.654	0.146	3.321	3.537	3.686	3.761	3.925
$\mathbb{E}_t[pe_{t,t+5}]$	72	3.603	0.284	2.864	3.408	3.590	3.803	4.208
q_t	72	0.596	0.236	0.211	0.408	0.587	0.731	1.202
U_t	72	0.061	0.021	0.036	0.046	0.054	0.078	0.130
θ_t	72	0.598	0.315	0.160	0.339	0.558	0.747	1.438
δ_t	72	0.350	0.058	0.265	0.316	0.354	0.370	0.689

Notes: This table reports summary statistics for ex-post realized outcomes (Actual), subjective expectations (Survey), and machine expectations (Machine) of key variables used in the variance decomposition. The forecasted variables are $h = 5$ year present discounted values of discount rates $r_{t,t+h}$, cash flows $e_{t,t+h}$, and price-earnings ratios $pe_{t,t+h}$, as defined in equation (17). Aggregate labor market variables include the vacancy filling rate q_t, unemployment rate U_t, vacancy-to-unemployment ratio θ_t, and job separation rate δ_t. Portfolio-level variables are constructed by aggregating employment and forecast data across firms within each book-to-market group, holding portfolio assignment fixed at the time of portfolio formation. Subjective expectations at the aggregate level $\mathbb{F}_t$ are based on survey forecasts from the CFO survey for stock returns and from IBES for earnings growth. Subjective expectations at the portfolio level $\mathbb{F}_t$ are based on survey forecasts from the IBES survey for both stock returns and earnings growth. Machine expectations $\mathbb{E}_t$ are based on forecasts from Long Short-Term Memory (LSTM) neural networks $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$, where parameters $\boldsymbol{\beta}_{h,t}$ are estimated in real time using $\mathcal{X}_t$, a large-scale dataset of macroeconomic, financial, and textual data. The sample is quarterly and spans 2005Q1 to 2023Q4.

OA.2 Variance Decompositions

OA.2.1 Baseline Specification

Table OA.2 reports a variance decomposition of the aggregate vacancy filling rate based on equation (22). Under objective expectations, discount rate fluctuations explain the largest share of variation, accounting for 69.1% at the five-year horizon. Under subjective expectations, cash flow beliefs dominate at all horizons, accounting for 96.7% in the five-year horizon.

Table OA.2: Time-Series Decomposition of the Vacancy Filling Rate

Horizon h (Years)	1	2	3	4	5
(a) Objective Expectations: $\log q_t = c_q + \mathbb{E}_t[r_{t,t+h}] - \mathbb{E}_t[e_{t,t+h}] - \mathbb{E}_t[pe_{t,t+h}]$					
Discount Rate	0.187***	0.309***	0.585***	0.653***	0.691***
t-stat	(3.310)	(4.708)	(5.977)	(6.974)	(6.659)
(-) Cash Flow	0.027	0.026	0.051	0.055	0.066
t-stat	(0.090)	(0.181)	(0.364)	(0.459)	(0.472)
(-) Price-Earnings	0.799***	0.720***	0.415***	0.331***	0.201**
t-stat	(5.620)	(4.322)	(3.332)	(2.845)	(1.716)
Residual	−0.013	−0.054	−0.051	−0.039	0.042
t-stat	(−0.030)	(−0.141)	(−0.076)	(−0.046)	(0.049)
N	76	76	76	76	76
(b) Subjective Expectations: $\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}]$					
Discount Rate	−0.007	−0.005	−0.019	−0.014	−0.010
t-stat	(−0.457)	(−0.130)	(−0.400)	(−0.157)	(−0.091)
(-) Cash Flow	0.325***	0.641***	0.717***	0.892***	0.967***
t-stat	(3.939)	(4.500)	(4.661)	(5.572)	(7.097)
(-) Price-Earnings	0.629***	0.366***	0.206***	0.068	0.028
t-stat	(8.383)	(4.231)	(2.896)	(0.701)	(0.313)
Residual	0.052	−0.002	0.096	0.054	0.015
t-stat	(0.186)	(−0.008)	(0.292)	(0.126)	(0.039)
N	76	76	76	76	76

Notes: This table reports variance decompositions of the aggregate vacancy filling rate under objective expectations (panel (a)) or subjective expectations (panel (b)). Each row reports the share of the variation in vacancy filling rates that can be explained by h-year expected present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$, as defined in equation (17). Residual term represents the variation in vacancy filling rates that are not captured by the other components. Positive numbers in the Cash Flow and Price-Earnings rows represent the negative of the regression coefficients, ensuring that all variance shares are positive and sum to unity. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Objective expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2023Q4. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

OA.2.2 Role of Model Misspecification and Approximation Errors

This section extends the variance decompositions by allowing for two residual sources: (*i*) approximation errors arising from the Campbell and Shiller (1988) log-linearization, and (*ii*) misspecification errors that stem from simplifying assumptions in the search model, such as ignoring firing costs or endogenous separations.

Campbell-Shiller residual For any horizon $h \geq 1$,

$$pe_t = \sum_{j=1}^{h} \rho^{j-1}\big(c_{pe} + \mathbb{F}_t[\Delta e_{t+j}] - \mathbb{F}_t[r_{t+j}]\big) + \rho^h \mathbb{F}_t[pe_{t+h}] + \upsilon^{CS}_{t,h}, \tag{OA.1}$$

where $\rho \in (0,1)$ is the log-linearization constant. $\upsilon^{CS}_{t,h}$ captures any approximation errors from the log-linearization and allows for the possibility that the Campbell-Shiller present value identity may be misspecified under subjective beliefs.

Search-model residual To allow for model misspecification in the hiring condition, such as layoff frictions or deviations from constant returns in production and matching, I add a separate residual $\upsilon^{M}_{t,h}$:

$$\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}] - \upsilon^{CS}_{t,h} - \upsilon^{M}_{t,h}, \tag{OA.2}$$

where $\upsilon^{M}_{t,h}$ captures deviations between the observed vacancy-filling rate and the theoretical expression implied by the baseline search model. For instance, if firing frictions create an option value of waiting to hire, firms may be more reluctant to fill vacancies than the baseline model suggests, an effect that would be captured by $\upsilon^{M}_{t,h}$.

Variance decomposition with residuals With both residuals included, the variance decomposition becomes

$$1 = \underbrace{\frac{\mathrm{Cov}(\mathbb{F}_t[r_{t,t+h}], \log q_t)}{\mathrm{Var}(\log q_t)}}_{\text{Discount Rate News}} - \underbrace{\frac{\mathrm{Cov}(\mathbb{F}_t[e_{t,t+h}], \log q_t)}{\mathrm{Var}(\log q_t)}}_{\text{Cash Flow News}} - \underbrace{\frac{\mathrm{Cov}(\mathbb{F}_t[pe_{t,t+h}], \log q_t)}{\mathrm{Var}(\log q_t)}}_{\text{Future Price–Earnings News}} - \underbrace{\frac{\mathrm{Cov}\left(\upsilon^{CS}_{t,h}, \log q_t\right)}{\mathrm{Var}(\log q_t)}}_{\text{Campbell–Shiller Residual}} - \underbrace{\frac{\mathrm{Cov}\left(\upsilon^{M}_{t,h}, \log q_t\right)}{\mathrm{Var}(\log q_t)}}_{\text{Search Model Residual}}.$$

The cross-sectional decomposition for $\widetilde{hl}_{i,t}$ from equation (7) includes analogous residuals $\widetilde{\upsilon}^{CS}_{i,t,h}$ and $\widetilde{\upsilon}^{M}_{i,t,h}$. In all figures and tables, we report the two residual components separately: the Campbell-Shiller residual reflects approximation noise from the log-linearization, while the model residual captures specification errors in the underlying search model.

Table OA.3 shows that both residual components are approximately orthogonal to the main decomposition terms, indicating that neither materially distorts the attribution. Time-series and cross-sectional correlations remain small, confirming that approximation and misspecification errors do not drive the main results.

Table OA.3: Correlation with Residual Terms from Regression Coefficients

Component	Campbell–Shiller Residual		Search Model Residual	
	Objective	Subjective	Objective	Subjective
(a) Time-Series				
Current Price–Earnings pe_t	0.001	0.007	0.010	0.170
Vacancy Filling Rate $\log q_t$	-0.003	0.010	-0.018	0.051
Discount Rate $\mathbb{F}_t[r_{t,t+5}]$	-0.002	0.009	-0.074	-0.155
Cash Flow $\mathbb{F}_t[e_{t,t+5}]$	-0.013	-0.002	0.046	0.048
Future Price–Earnings $\mathbb{F}_t[pe_{t,t+5}]$	-0.002	0.031	-0.024	-0.127
(b) Cross-Section				
Current Price–Earnings $pe_{i,t}$	-0.001	-0.000	0.005	0.001
Vacancy Filling Rate $\log q_{i,t}$	0.000	-0.002	-0.036	-0.128
Discount Rate $\mathbb{F}_t[r_{i,t,t+5}]$	-0.002	0.014	-0.079	-0.112
Cash Flow $\mathbb{F}_t[e_{i,t,t+5}]$	-0.022	-0.002	-0.033	0.034
Future Price–Earnings $\mathbb{F}_t[pe_{i,t,t+5}]$	0.002	0.053	0.014	0.130

Notes: This table reports correlations between residuals and decomposition components under both objective and subjective beliefs. The Campbell–Shiller residual ($\upsilon^{CS}_{t,h}$) captures log-linearization and measurement errors in the price–earnings decomposition. The search-model residual ($\upsilon^{M}_{t,h}$) captures deviations from the baseline search model, such as omitted firing costs or endogenous separations. Time-series correlations use aggregate data; cross-sectional correlations use firm-level deviations from time-t means.

OA.2.3 Belief Distortions and Vacancy Filling Rate

To directly quantify the importance of belief distortions in subjective beliefs, I consider predictive regressions of belief distortions in subjective expectations of discount rates, cash flows, and price-earnings ratios on the vacancy filling rate. I define the belief distortion as the difference between subjective and machine expectations. Table OA.4 reports estimates $\beta_{1,B}$ from regressing belief distortions in subjective discount rate, cash flow, and log price-earnings expectations on the vacancy filling rate:

$$\mathbb{F}_t[y_{t+h}] - \mathbb{E}_t[y_{t+h}] = \beta_{0,B} + \beta_{1,B} \log q_t + \varepsilon_{t,B}, \qquad y = r, e, pe$$

The results indicate that distortions in survey forecasts are important contributors to fluctuations in vacancy filling rates, especially at longer horizons. At the five-year horizon, distortions in cash flow expectations lead survey respondents to over-weight 90.1% of the variation in vacancy filling rates to the cash flow component. This mis-perception is counteracted by distortions in subjective discount rate expectations, which leads survey respondents to under-weight 70.1% of the variation in the vacancy filling rate. These findings emphasize the importance of belief distortions in driving labor market fluctuations. The profile of the response across forecast horizons is broadly consistent with the profile of the MSE ratios across horizons in Figure 2. For discount rate and cash flow expectations, the machine outperformed the survey by a wider margin over longer horizons, suggesting that the belief distortions in survey responses likely play a bigger role over these longer horizons.

Table OA.4: Belief Distortions in Subjective Beliefs and the Vacancy Filling Rate

Horizon h (Years)	1	2	3	4	5
Belief Distortions: $\mathbb{F}_t[y_{t+h}] - \mathbb{E}_t[y_{t+h}] = \beta_{0,B} + \beta_{1,B} \log q_t + \varepsilon_{t,B}, \quad y = r, e, pe$					
Discount Rate	-0.194	-0.313^{**}	-0.604^{***}	-0.667^{***}	-0.701^{***}
t-stat	(-1.574)	(-2.167)	(-2.896)	(-2.918)	(-2.740)
(-) Cash Flow	0.299	0.615^{***}	0.666^{***}	0.837^{***}	0.901^{***}
t-stat	(1.421)	(5.476)	(5.703)	(7.365)	(6.665)
(-) Price-Earnings	-0.170	-0.354^{**}	-0.209	-0.262	-0.174
t-stat	(-0.464)	(-2.373)	(-0.503)	(-0.479)	(-0.292)
Residual	-0.065	-0.052	-0.147	-0.093	0.026
t-stat	(-0.148)	(-0.219)	(-0.306)	(-0.154)	(0.040)
N	76	76	76	76	76

Notes: This table reports estimates $\beta_{1,B}$ from regressing the survey belief distortion $\mathbb{F}_t[y_{t+h}] - \mathbb{E}_t[y_{t+h}]$ on the vacancy filling rate q_t. y_{t+h} denotes the dependent variable of type j to be predicted h years ahead of time t. The components of the decomposition are h-year present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$. The residual term captures variation in the vacancy filling rate that cannot be explained by the three components. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts from the CFO survey for stock returns, and IBES for earnings growth. Machine expectations are based on machine learning forecasts $\mathbb{E}_t$ from Long Short-Term Memory (LSTM) neural networks $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$, whose parameters $\boldsymbol{\beta}_{h,t}$ are estimated in real time using $\mathcal{X}_t$, a large scale dataset of macroeconomic, financial, and textual data. The belief distortion is defined as the difference between subjective and machine expectations: $\mathbb{F}_t - \mathbb{E}_t$. The sample is quarterly from 2005Q1 to 2023Q4. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

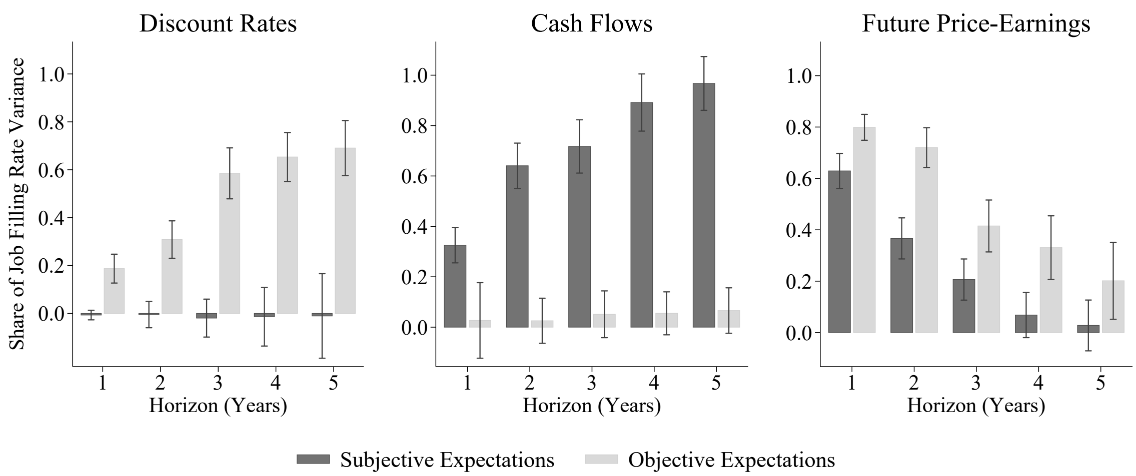
OA.2.4 First Differences

The decomposition in equation (22) may be more accurate in first differences than in levels, as low-frequency variation in the vacancy filling rate or subjective expectations can introduce measurement error. This concern is similar to the argument in Cochrane (1991), who points to low-frequency changes in fundamentals as a potential source of measurement error in the context of the q-theory of investment. Figure OA.1 estimates the variance decomposition of the vacancy filling rate from equation (22) in first differences:

$$\Delta \log q_t = \Delta \mathbb{E}_t[r_{t,t+h}] - \Delta \mathbb{E}_t[e_{t,t+h}] - \Delta \mathbb{E}_t[pe_{t,t+h}]$$
$$\Delta \log q_t = \Delta \mathbb{F}_t[r_{t,t+h}] - \Delta \mathbb{F}_t[e_{t,t+h}] - \Delta \mathbb{F}_t[pe_{t,t+h}]$$

Under objective expectations, discount rate fluctuations explain the largest share of variation, accounting for 58.7% at the five-year horizon. Under subjective expectations, cash flow beliefs dominate, accounting for 90.6% at the five-year horizon.

Figure OA.1: Variance Decomposition of Vacancy Filling Rate: First Differences



Notes: Figure reports variance decompositions of the aggregate vacancy filling rate in first differences. Each panel reports the share of the variation in vacancy filling rates that can be explained by h-year expected present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$, as defined in equation (17). Light (dark) bars show the contribution under objective (subjective) expectations. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Objective expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4 quarters.

OA.2.5 VAR Estimates

To validate the robustness of the variance decompositions, I estimate a Vector Autoregression (VAR) for the log vacancy filling rate $\log q_t$ and its forward-looking components under subjective or objective expectations. For the case of subjective beliefs, the VAR is estimated using survey expectations for future returns, earnings growth, and price-earnings ratios:

$$X_{t+1} = AX_t + \varepsilon_{t+1}, \quad X_t = [\mathbb{F}_t[r_{t,t+1}] \quad \mathbb{F}_t[e_{t,t+1}] \quad \mathbb{F}_t[pe_{t,t+1}] \quad \log q_t]'.$$

From the theoretical framework in Section 4, the log vacancy filling rate can be decomposed as:

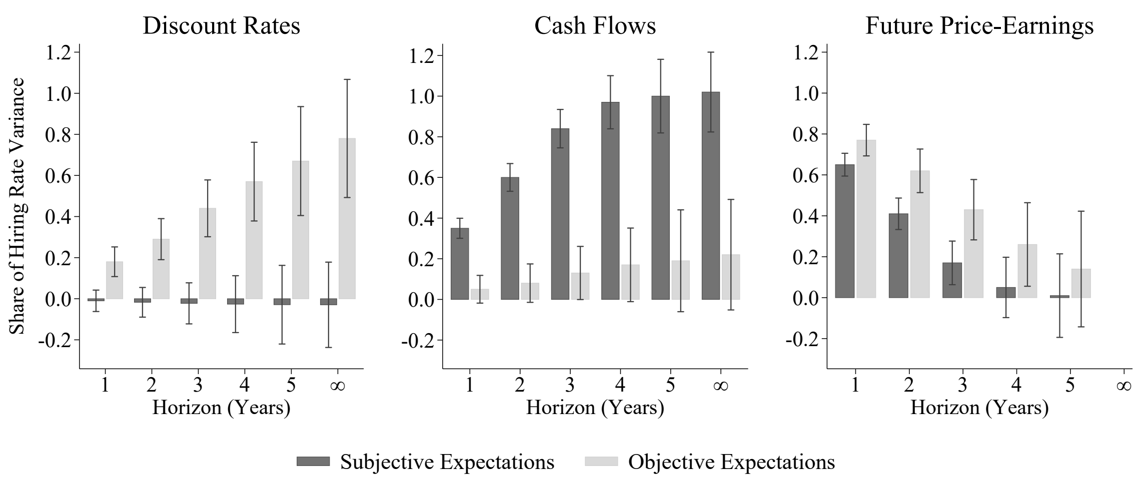
$$\log q_t = c_q + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \rho^h \mathbb{F}_t[pe_{t+h}]$$

where the expected present values $\mathbb{F}_t[r_{t,t+h}]$ and $\mathbb{F}_t[e_{t,t+h}]$ are constructed recursively using the VAR forecast. As $h \to \infty$, the terminal value $\rho^h \mathbb{F}_t[pe_{t+h}]$ converges to zero under a transversality condition, yielding the long-run decomposition:

$$\log q_t = c_q + \mathbb{F}_t[r_{t,t+\infty}] - \mathbb{F}_t[e_{t,t+\infty}].$$

The same procedure is repeated using machine learning forecasts $\mathbb{E}_t[\cdot]$ to obtain the decomposition under objective expectations. Figure OA.2 reports variance shares across horizons $h = 1$ to $h = 5$, as well as the full-horizon case $h = \infty$. Under objective expectations, discount rate fluctuations explain an increasing share of variation, rising to 78.1% at long horizons. Under subjective expectations, cash flow beliefs dominate at all horizons, accounting for 102.0% in the long run.

Figure OA.2: Variance Decomposition of Vacancy Filling Rate: VAR Estimates



Notes: Figure reports variance decompositions of the aggregate vacancy filling rate based on a Vector Autoregression (VAR). Each panel reports the share of the variation in vacancy filling rates that can be explained by h-year expected present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$, as defined in equation (17). Light (dark) bars show the contribution under objective (subjective) expectations. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Objective expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows bootstrapped 95% confidence intervals.

OA.2.6 Alternative Survey Measures of Subjective Discount Rates

The small role played by subjective discount rate expectations in explaining the vacancy filling rate holds more generally across alternative survey forecasts of stock returns. Table OA.5 reports estimates from regressing 1 year ahead survey expectations of stock returns $\mathbb{F}_t[r_{t,t+h}]$ on the log vacancy filling rate q_t under alternative survey forecasts of stock returns. In all survey measures, the estimates suggest a weak relationship between subjective stock return expectations $\mathbb{F}_t^s[r_{t,t+h}]$ and the vacancy filling rate q_t.

$r_{t,t+h}$ denotes h year CRSP stock returns (with dividends) or S&P 500 price growth from time t to $t+h$, depending on the concept that survey respondents are asked to predict: log stock returns for CB, SOC, Gallup/UBS, and CFO; log price growth for Livingston. $\mathbb{F}_t^s[r_{t,t+h}]$ denotes subjective expectations of stock returns or price growth from survey s. CoC and Hurdle denotes corporate cost of capital and hurdle rates constructed in Gormsen and Huber (2025). The forecast horizon has been limited to 1 year ahead due to limited data availability in the alternative surveys. The sample is quarterly over 2005Q1 to 2023Q4 when considering the NX, CB, SOC, and CFO surveys, 2005Q1 to 2008Q4 for Gallup/UBS, and semi-annual over 2005Q1 to 2023Q4 from Q2 and Q4 of each calendar year for Livingston.

To summarize the alternative survey measures into a single series, the Filtered Invesotr (FI) series extracts the common component of subjective discount rates using a Kalman filter. The state variable is a latent h-month ahead expected stock return capturing investors' subjective beliefs $S_t \equiv \mathbb{F}_t[r_{t+h}]$, which evolves according to an AR(1) state equation $S_t = C(\Theta) + T(\Theta)S_{t-1} + R(\Theta)\varepsilon_t$, where C, T, R are matrices of the model's primitive parameters $\Theta = (\alpha, \rho, \sigma_\varepsilon)'$. ε_t is an innovation to the latent expectation that was unpredictable from the point of view of the forecaster. α is the intercept, ρ is the persistence, and σ_ε is the standard deviation of the latent innovation error. The Observation equation takes the form $X_t = D + ZS_t + Uv_t$, where $h = 12$ months is a fixed forecast horizon. The observation vector X_t contains measures of survey expected returns listed above over the next h periods. v_t is a vector of observation errors with standard deviations in the diagonal matrix U. Z and D are parameters that have been set to 1s and 0s, respectively. I use the Kalman filter to estimate the remaining parameters $\alpha, \rho, \sigma_\varepsilon, U$. Since some of our observable series are not available at all frequencies and/or over the full sample, the state-space estimation fills in missing values using the Kalman filter.

Table OA.5: Variance Decomposition of Vacancy Filling Rate: Alternative Discount Rates

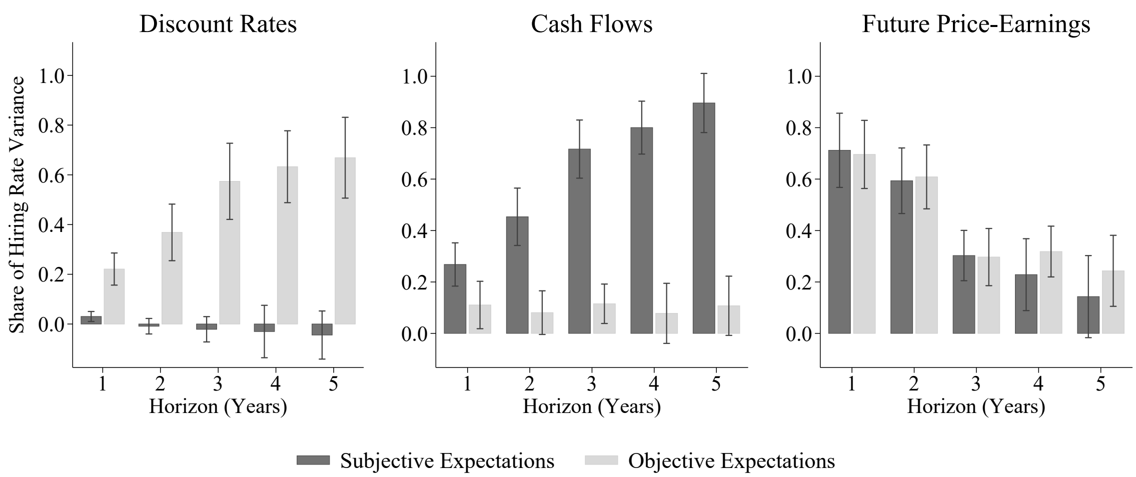
Horizon h (Years)	1	1	1	1	1	1	1	1
	Subjective Expectations: $\log q_t = c_q + \mathbb{F}_t^s[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}]$							
Survey s	FI	NX	CB	SOC	Gallup	Liv	CoC	Hurdle
Discount Rate	0.013	−0.011	0.026	0.002	−0.065	0.067	0.024	0.013
t-stat	(0.614)	(−0.249)	(0.504)	(0.103)	(−0.922)	(0.181)	(0.734)	(0.522)
Adj. R^2	0.070	0.012	0.069	0.009	0.216	0.045	0.232	0.154
N	76	76	76	76	16	40	76	76

Notes: Table reports slope (β_1) estimates from regressing $h = 1$ year ahead survey expectations of stock returns $\mathbb{F}_t[r_{t,t+h}]$ on the log vacancy filling rate q_t. $r_{t,t+h}$ denotes h year CRSP stock returns (with dividends) or S&P 500 price growth from time t to $t+h$, depending on the concept that survey respondents are asked to predict: log stock returns for CB, SOC, Gallup/UBS, and CFO; log price growth for Livingston. $\mathbb{F}_t^s[r_{t,t+h}]$ denotes subjective expectations of stock returns or price growth from survey s. CoC and Hurdle denotes corporate cost of capital and hurdle rates constructed in Gormsen and Huber (2025). Filtered Investor (FI) expectations summarize the alternative survey measures into a single series using a Kalman filter. The sample is quarterly over 2005Q1 to 2023Q4 when considering the NX, CB, SOC, and CFO surveys, 2005Q1 to 2008Q4 for Gallup/UBS, and semi-annual over 2005Q1 to 2023Q4 from Q2 and Q4 of each calendar year for Livingston. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

OA.2.7 Extended Historical Sample

Figure OA.3 reports the variance decomposition of the vacancy filling rate from equation (22) using an extended quarterly sample from 1983Q4 to 2023Q4. Subjective cash flow expectations are measured using IBES survey forecasts of earnings growth, available from 1983Q4. Subjective discount rate expectations are extended by extracting a common latent component from multiple historical survey sources from Table OA.5 using a state-space model estimated via the Kalman filter. The latent state S_t is interpreted as the one-year-ahead expected stock return, $\mathbb{F}_t[r_{t+1}]$. To construct the five annual forecasts needed for the present-value sum $r_{t,t+h}$, I impose a flat term-structure assumption and set $\mathbb{F}_t[r_{t+j}] = S_t$ for $j = 1, \ldots, 5$. This approach ensures that all horizons are anchored by the common latent factor while remaining consistent with the information set of the historical surveys. The extended sample results are consistent with the baseline. Under objective expectations, discount rate fluctuations explain 66.9% of vacancy filling rate variation at the five-year horizon. Under subjective expectations, distorted cash flow beliefs dominate, accounting for 89.6%.

Figure OA.3: Variance Decomposition of Vacancy Filling Rate: Extended Sample 1983Q4–2023Q4



Notes: Figure reports variance decompositions of the aggregate vacancy filling rate using an extended sample from 1983Q4 to 2023Q4. Each panel reports the share of the variation in vacancy filling rates that can be explained by h-year expected present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$, as defined in equation (17). Light (dark) bars show the contribution under objective (subjective) expectations. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Objective expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. Each bar shows Newey-West 95% confidence intervals with lags = 4 quarters.

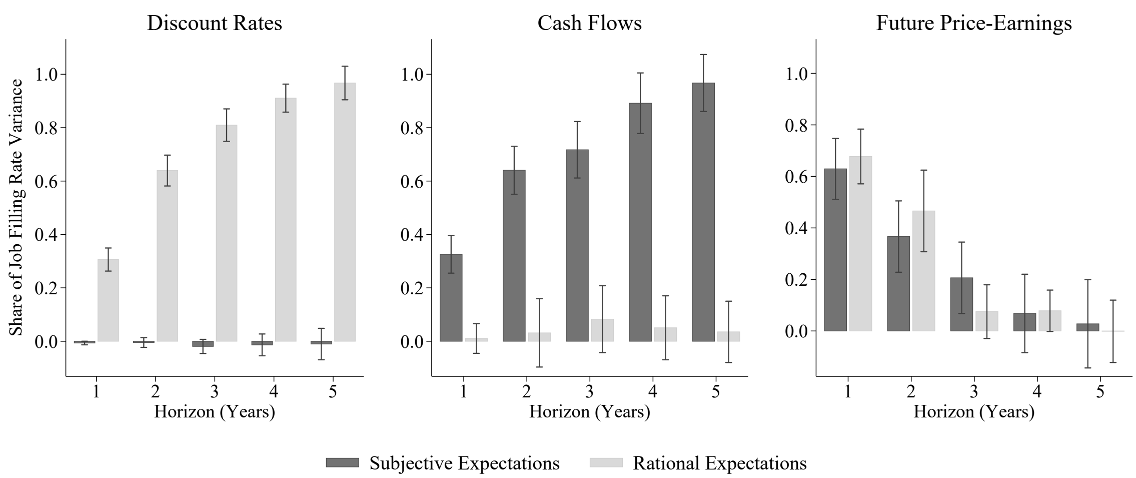
OA.2.8 Ex-post Decomposition

Since the log-linear decomposition of the vacancy filling rate holds both ex-ante and ex-post, a variance decomposition of the vacancy filling rate can also be estimated using ex-post realized data, under the assumption of the firm's perfect foresight:

$$1 \approx \underbrace{\frac{Cov\left[r_{t,t+h}, \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Discount Rate news}} - \underbrace{\frac{Cov\left[e_{t,t+h}, \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Cash Flow News}} - \underbrace{\frac{Cov\left[pe_{t,t+h}, \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Future Price-Earnings News}}$$

Table OA.4 reports the estimates. For the main sample covering 2005Q1 to 2023Q4, at the 5 year horizon, 79.4% of the variation in the vacancy filling rate is driven by discount rate news. In contrast, cash flow news has a smaller effect, contributing only 10.3% over the same period. For the full sample covering 1965Q1 to 2023Q4, at the 5 year horizon, 78.6% of the variation in the vacancy filling rate is driven by discount rate news. In contrast, cash flow news has a smaller effect, contributing only 9.5% over the same period.

Figure OA.4: Variance Decomposition of Vacancy Filling Rate: Ex-Post Measure 1965Q1–2023Q4

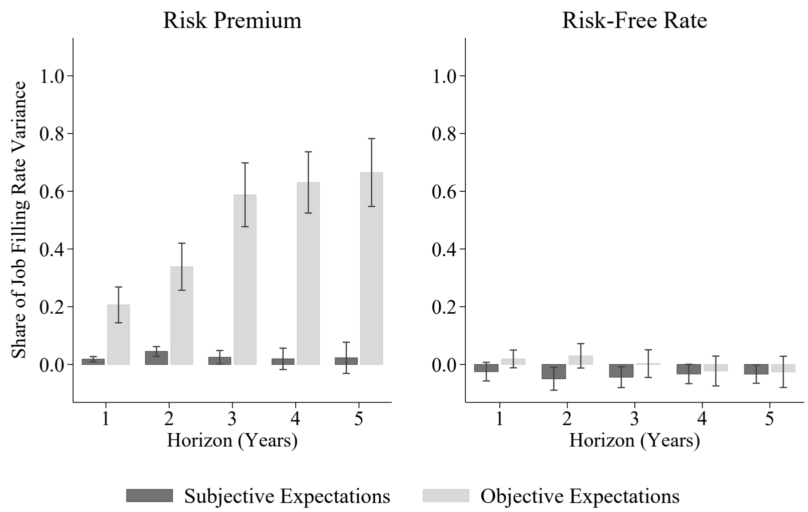


Notes: Figure reports variance decompositions of the vacancy filling rate from equation using ex-post realized outcomes. Each panel reports the share of the variation in vacancy filling rates that can be explained by h-year expected present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$, as defined in equation (17). Light bars show the contribution under objective expectations. The sample is quarterly from 1965Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4 quarters.

OA.2.9 Risk Premia vs. Risk-Free Rate

Risk-free rates play only a small role in explaining fluctuations in vacancy filling rates. Figure OA.5 plots estimates from regressing subjective expectations implied by forecasts from the Survey of Professional Forecasters (SPF), and machine expectations of h year ahead annualized log 3-month Treasury bill rates on the vacancy filling rate. Under all measures of beliefs and all horizons considered, the contribution from risk-free rates explain less than 5% of the variation in vacancy filling rates. The result suggests that the significant contribution of objective discount rates in Table OA.2 is driven by fluctuations in risk premia instead of risk-free rates.

Figure OA.5: Variance Decomposition of Vacancy Filling Rate: Risk Premia vs. Risk-Free Rate



Notes: Figure plots estimates from regressing h year present discounted value of annualized log 3-month Treasury bill rates $\sum_{j=1}^{h} \rho^{j-1} r_{t+j}^{f}$ on the vacancy filling rate under alternative assumptions about the firm's beliefs. Subjective expectations $\mathbb{F}_t$ of risk-free rates are based on survey forecasts from the Survey of Professional Forecasters. Subjective expectations of the equity risk premium is defined as the difference between CFO survey S&P 500 stock return forecast and the SPF risk-free rate forecast. Machine expectations are based on machine learning forecasts $\mathbb{E}_t$ from Long Short-Term Memory (LSTM) neural networks $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$, whose parameters $\boldsymbol{\beta}_{h,t}$ are estimated in real time using $\mathcal{X}_t$, a large scale dataset of macroeconomic, financial, and textual data. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

OA.2.10 Additional Controls

The large contribution from subjective long-term cash flow expectations in explaining the vacancy filling rate is robust to conditioning on additional variables that could distort the relationship. Table OA.6 re-estimates the subjective variance decomposition at the 5 year horizon with additional control variables on the right-hand side of the regression: 1 year lag of the log vacancy filling rate and the dependent variable, and the 1 year ahead survey forecast of the same variable. Controlling for the short-term expectation $\mathbb{F}_t[y_{t+1}]$ accounts for the possibility that survey respondents' long-term forecasts could be influenced by the short-term component of cash flows (Nagel and Xu, 2021).

Table OA.6: Variance Decomposition of Vacancy Filling Rate: Additional Controls

Dep. Var.	Discount Rate	(-) Cash Flow	(-) Price-Earnings
Horizon h (Years)	5	5	5
Subjective Expectations: $\mathbb{F}_t[y_{t+h}] = \beta_{0,\mathbb{F}} + \beta_{1,\mathbb{F}} \log q_t + \beta_{2,\mathbb{F}} \log q_{t-1} + \beta_{3,\mathbb{F}}\mathbb{F}_{t-1}[y_{t+h-1}] + \beta_{4,\mathbb{F}}\mathbb{F}_t[y_{t+1}] + \varepsilon_{t,\mathbb{F}}$			
Share of job filling rate variation	-0.007	0.855^{***}	0.049
t-stat	(-0.108)	(4.865)	(0.455)
Adj. R^2	0.456	0.514	0.533
N	76	76	76
Controls	Yes	Yes	Yes

Notes: Table reports variance decompositions of the vacancy filling rate under subjective expectations $\mathbb{F}_t$ implied by survey forecasts. y_{t+h} denotes the dependent variable of type j to be predicted $h = 5$ years ahead of time t: h year present discounted values of discount rates ($r_{t,t+h} = \sum_{j=1}^{h} \rho^{j-1} r_{t+j}$), cash flows ($e_{t,t+h} = el_t + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{t+j}$), and log price-earnings ratios ($pe_{t,t+h} = \rho^h pe_{t+h}$). Subjective expectations $\mathbb{F}_t$ are based on survey forecasts from the CFO survey for stock returns, and IBES for earnings growth. The sample is quarterly over 2005Q1 to 2023Q4. Newey-West corrected t-statistics with lags = 4 are reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

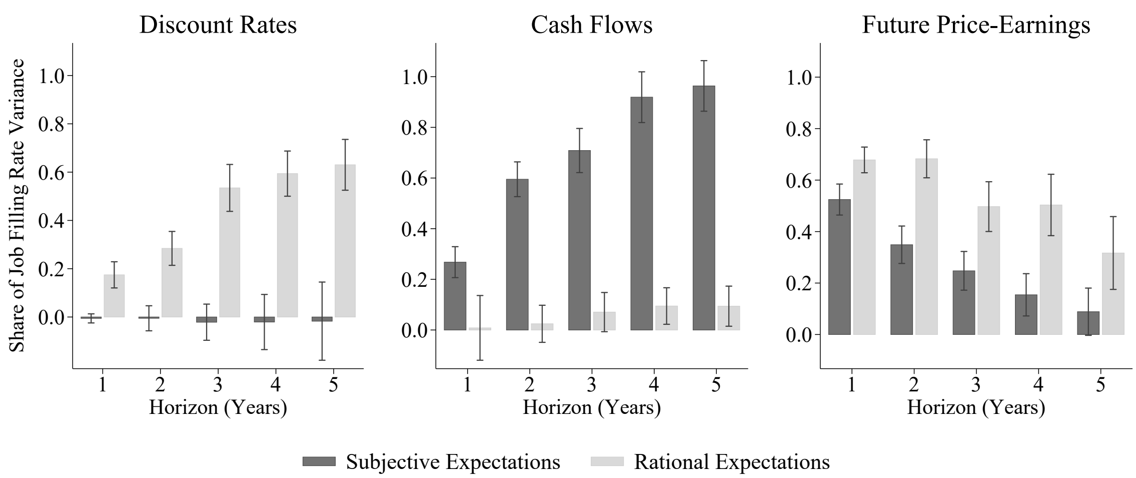
OA.2.11 All Listed Firms

Figure OA.6 reports the variance decomposition of the vacancy filling rate from equation (22) using an expanded definition of subjective expectations that includes all publicly listed firms with IBES analyst coverage, rather than restricting to the S&P 500. Subjective cash flow expectations are computed as value-weighted aggregates of IBES median forecasts of long-horizon earnings growth across all covered firms. Subjective discount rate expectations are constructed analogously, using the same survey-based measures as in the baseline but applying the expanded firm universe for consistency in coverage.

The results are similar to the baseline. Under rational expectations, discount rate fluctuations explain 63.0% of the variation in vacancy filling rates at the five-year horizon, while under subjective expectations, distorted cash flow beliefs remain dominant, accounting for 96.3%. The similarity in results suggests that the dominance of cash flow distortions under subjective beliefs is not specific to large-cap firms in the S&P 500 but holds more broadly across publicly listed firms with analyst coverage.

While this paper focuses on publicly listed firms due to data limitations, preliminary evidence suggest that similar patterns likely emerge among smaller private businesses. A 2010 report from the National Federation of Independent Business (NFIB) on small business credit during the recession shows that hiring decisions were primarily driven by pessimism about future sales rather than financing constraints. At the time, 51% of small employers cited weak sales expectations as their top concern, compared to just 8% who cited access to credit (Dennis, 2010). To the extent that access to credit capture financial frictions that would show up in discount rates, this survey suggests that subjective beliefs about future cash flows also shape employment decisions in the small business sector.

Figure OA.6: Variance Decomposition of Vacancy Filling Rate: All Listed Firms



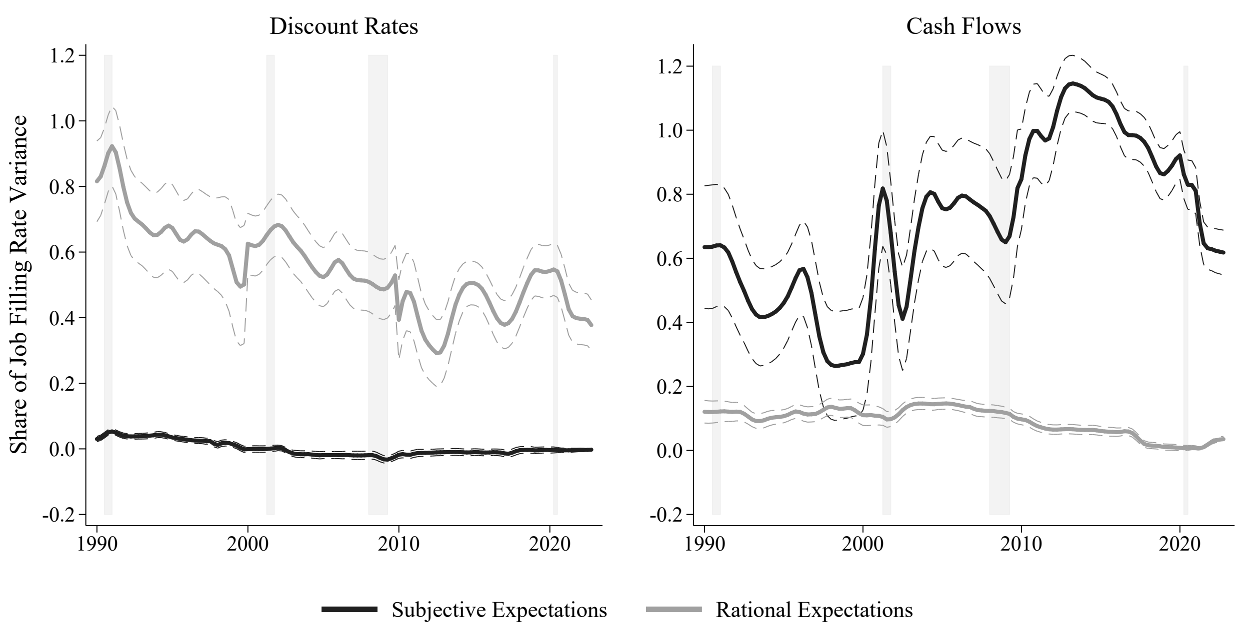
Notes: Figure reports variance decompositions of the aggregate vacancy filling rate using forecasts aggregated over all publicly listed firms with IBES analyst coverage. Each panel reports the share of the variation in vacancy filling rates that can be explained by h-year expected present discounted values of discount rates $r_{t,t+h}$, (negative) cash flows $e_{t,t+h}$, and (negative) price-earnings ratios $pe_{t,t+h}$, as defined in equation (17). Light (dark) bars show the contribution under rational (subjective) expectations. Subjective expectations $\mathbb{F}_t$ are based on IBES survey forecasts of financial analysts aggregated over all covered firms. Rational expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4 quarters.

OA.2.12 Time-Varying Parameters

Figure OA.7 estimates time-series variance decompositions of the vacancy filling rate over rolling samples of trailing 15-year windows. The estimated rational discount rate component is large and the rational cash flow component is small throughout the rolling samples. In contrast, the subjective discount rate component is small and the subjective cash flow component is large throughout the rolling samples. The persistent dominance of subjective cash flow expectations across all time periods confirms that belief distortions are not episodic phenomena but represent and enduring features of expectation formation.

Nevertheless, there is notable variation in the estimated components over time, as the subjective cash flow component shows large increases during recessions. The sharp increases in the subjective cash flow component during recession periods indicate that firms respond to economic downturns by becoming excessively pessimistic about future cash flows. The rational discount rate component exhibits a gradual decline over the sample period, potentially reflecting structural changes in risk premia or monetary policy regimes.

Figure OA.7: Time-Series Decomposition of Vacancy Filling Rate: Time-Varying Parameters



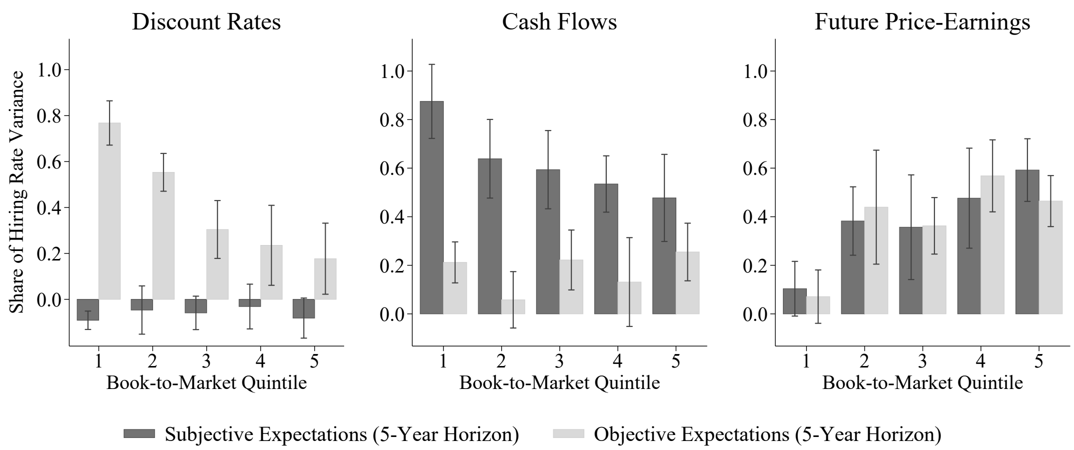
Notes: Figure estimates time-series decomposition of the vacancy filling rate over rolling samples of trailing 15-year windows. Grey line show the contribution under rational expectations. Dark line show the contribution under subjective expectations. Each dashed line shows Newey-West 95% confidence intervals with lags = 4. NBER recessions are shown with light gray shaded bars.

OA.2.13 Time-Series Decomposition of Hiring Rate by Book-to-Market and Size Portfolios

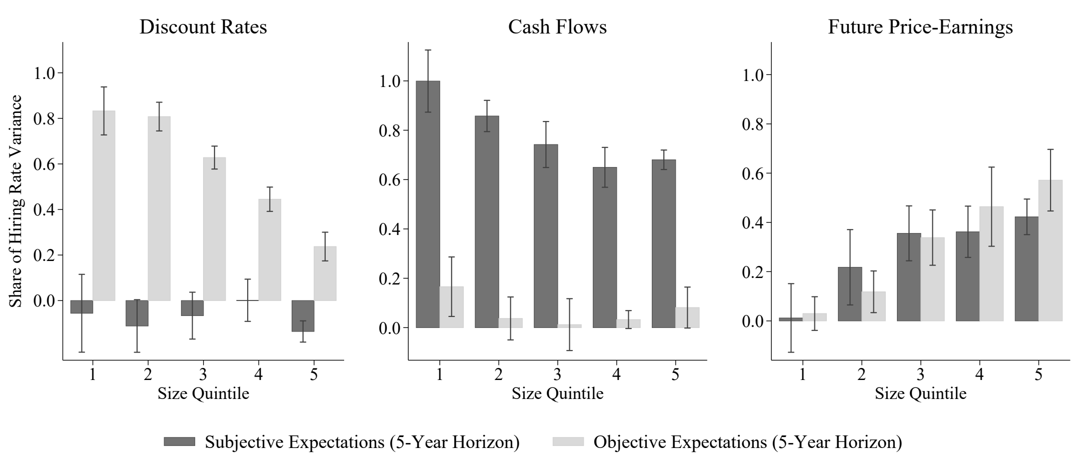
Figure OA.8 shows that belief distortions play a significant role in explaining the cross-sectional variation in hiring across book-to-market portfolios (panel (a)) and size portfolios (panel (b)). I run the time-series decomposition of the hiring rate separately for each of the portfolios. The decomposition reveals that under subjective expectations, distorted beliefs about future cash flows account for a larger share of hiring rate variation, particularly among low book-to-market (growth) firms and small firms. This pattern is consistent with the idea that growth firms and small firms are more sensitive to subjective beliefs about long-term fundamentals, amplifying the role of distorted expectations in their hiring decisions. In contrast, for high book-to-market (value) firms and large firms, the contribution of cash flow expectations remains relatively stable across subjective and objective benchmarks, suggesting their hiring is less exposed to belief distortions.

Figure OA.8: Time-Series Decomposition of Hiring Rate by Portfolio

(a) By Book-to-Market Portfolio



(b) By Size Portfolio

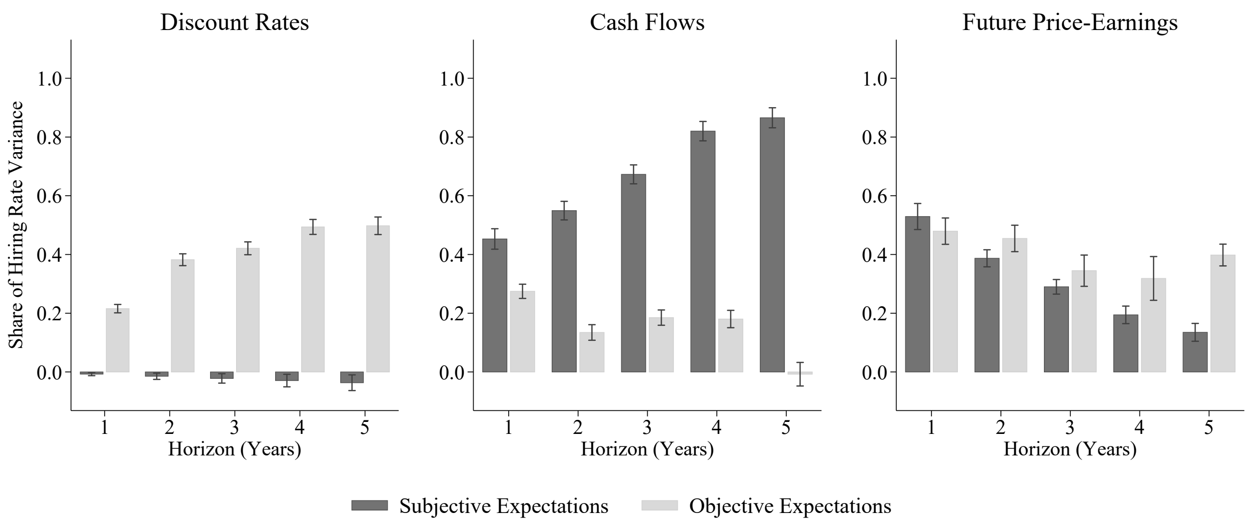


Notes: Figure estimates time-series decomposition of hiring rate separately for each of the five book-to-market (panel (a)) and size (panel (b)) portfolios. Firms have been sorted into five value-weighted portfolios by book-to-market ratio or size (market capitalization). Light bars show contributions under objective expectations; dark bars show contributions under subjective expectations. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

OA.2.14 Cross-Sectional Decomposition of Hiring Rate: By Industry

Figure OA.9 shows that belief distortions play a significant role in explaining the cross-sectional variation in hiring across Fama-French 49 industry portfolios.

Figure OA.9: Cross-Sectional Decomposition of Hiring Rate: By Industry



Notes: Figure estimates a cross-sectional decomposition of the hiring rate across Fama-French 49 industry portfolios. Light bars show contributions under objective expectations; dark bars show contributions under subjective expectations. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

OA.3 On-the-Job Search

The baseline model assumes that all hires come from the pool of unemployed workers. However, measured earnings and hiring flows reflect contributions from both unemployed-to-employed (UE) and job-to-job (J2J) transitions. To better capture the sources of observed hiring, this section extends the baseline model to allow for on-the-job search. This modification draws on recent work modeling labor market flows with job-to-job transitions (Kuhn et al., 2021; Faberman et al., 2022). Let a fraction ϕ of employed workers search for jobs each period, in addition to the unemployed. The total number of searchers is:

$$S_t = U_t + \phi L_t = U_t + \phi(1 - U_t), \tag{OA.3}$$

where U_t is the unemployment rate and $L_t = 1 - U_t$ is the employment rate. Vacant firms post V_t vacancies, and matches form via a constant returns to scale matching function $\mathcal{M}(S_t, V_t)$. Not all on-the-job searchers who receive an offer accept it. Let $\chi \in (0, 1)$ denote the fraction of employed searchers who accept a job offer. The effective hiring efficiency from the firm's perspective is:

$$\varphi_t = \frac{U_t + \chi\phi(1 - U_t)}{U_t + \phi(1 - U_t)}. \tag{OA.4}$$

The law of motion for employment becomes:

$$L_{t+1} = (1 - \delta_t)L_t + q_t\varphi_t V_t, \tag{OA.5}$$

where δ_t is the separation rate and $q_t = \frac{\mathcal{M}(S_t,V_t)}{V_t}$ is the vacancy filling rate. The Bellman equation for the firm's value is updated to reflect turnover due to J2J transitions:

$$\mathcal{V}(A_t, L_t) = \max_{V_t, L_{t+1}} \left\{ E_t + (1 - \phi\chi f_t)\mathbb{F}_t \left[M_{t+1}\mathcal{V}(A_{t+1}, L_{t+1}) \right] \right\}, \tag{OA.6}$$

subject to the employment accumulation equation above. The term $1 - \phi\chi f_t$ reflects the retention rate, accounting for voluntary separations from employed workers who successfully switch jobs. Under constant returns to scale, the firm's optimal vacancy posting condition implies:

$$\frac{\kappa}{q_t\varphi_t} = (1 - \phi\chi f_t) \cdot \frac{P_t}{L_{t+1}}, \tag{OA.7}$$

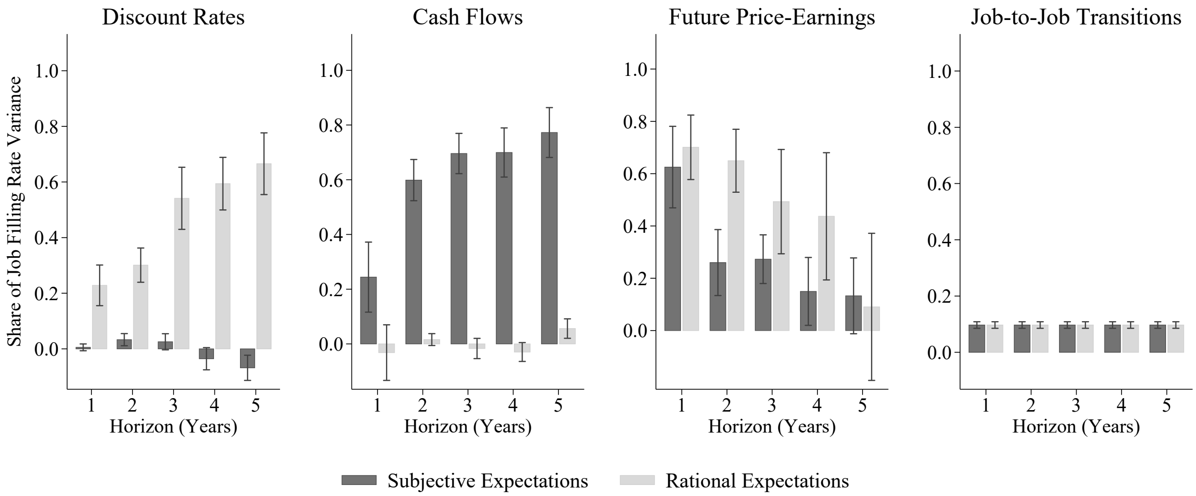
where $P_t = \mathbb{F}_t \left[M_{t+1}\mathcal{V}(A_{t+1}, L_{t+1}) \right]$ is the ex-dividend firm value and κ is the flow cost of posting a vacancy. Taking logs and rearranging, the log vacancy filling rate can be written as:

$$\log q_t = c_q - \log(1 - \phi\chi f_t) + \mathbb{F}_t[r_{t,t+h}] - \mathbb{F}_t[e_{t,t+h}] - \mathbb{F}_t[pe_{t,t+h}], \tag{OA.8}$$

where $c_q = \log\kappa - \log\varphi_t - \frac{c_{pe}(1-\rho^h)}{1-\rho}$ is a constant, $r_{t,t+h}$ is the present value of expected discount rates, $e_{t,t+h}$ is expected cumulative earnings growth, and $pe_{t,t+h}$ is the expected terminal price-earnings ratio. This decomposition extends the Campbell and Shiller (1988) present value identity to account for hiring frictions due to job-to-job transitions. The vacancy filling rate q_t is computed as the ratio of total hires to vacancies $q_t = \frac{H_t}{V_t}$ using JOLTS data for hires and job openings. The total search pool S_t includes both unemployed and a fraction $\phi = 0.12$ of employed workers, based on estimates from Kuhn et al. (2021) and Faberman et al. (2022). The job finding rate is then inferred from the matching function as $f_t = q_t \cdot \theta_t$, where labor market tightness is defined as $\theta_t = \frac{V_t}{S_t} = \frac{V_t}{U_t+\phi(1-U_t)}$. I assume that $\chi = 0.75$ of employed job seekers accept offers. These parameter values imply an endogenous efficiency term φ_t and a retention rate $1 - \phi\chi f_t$, which are used to adjust the firm's hiring incentives and derive the decomposition. Subjective expectations of earnings growth are from IBES, which aggregates analyst forecasts of total firm earnings and therefore reflect both UE and J2J hires.

Figure OA.10 presents the decomposition of the vacancy filling rate under this extended model with on-the-job search. Consistent with the baseline analysis, the cash flow component remains the dominant driver of variation in the vacancy filling rate under subjective expectations. However, accounting for job-to-job transitions modestly shifts the decomposition: the log retention rate term $\log(1 - \phi\chi f_t)$ explains 8.9% of the variation in $\log q_t$. This adjustment reflects the influence of selective separations on firms' incentives to post vacancies. Overall, the results reinforce the finding that distorted cash flow expectations are the primary driver of hiring fluctuations. The extension confirms that even when allowing for endogenous separations due to on-the-job search, subjective belief distortions about firm-level earnings continue to dominate the variation in hiring behavior.

Figure OA.10: Time-Series Decomposition of the Vacancy Filling Rate: On-the-Job Search



Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components of the time-series decomposition of the aggregate vacancy filling rate. Light bars show contributions under objective expectations; dark bars show contributions under subjective expectations. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

OA.4 Decreasing Returns to Scale and Composition Effects

Stock market valuations reflect average profits, while hiring decisions depend on marginal profits (Borovičková and Borovička, 2017). Decreasing returns to scale can amplify unemployment fluctuations even under a rational framework by making the marginal value of hiring more sensitive to productivity shocks, prompting firms to adjust vacancies more aggressively in response (Elsby and Michaels, 2013; Kaas and Kircher, 2015). Allowing for decreasing returns to scale introduces the notion of firm size. Changes in the equilibrium firm size distribution can thus introduce a composition effect that also contributes to fluctuations in the vacancy filling rate (Solon et al. (1994)).

This section relaxes the constant returns to scale (CRS) assumption by allowing for decreasing returns to scale (DRS) in the production function. Assume that firm i's output is $Y_{i,t} = F(L_{i,t}) = A_{i,t}L_{i,t}^{\alpha}$, where $A_{i,t}$ is an exogenous productivity process and $0 < \alpha < 1$. This introduces a "DRS wedge" between marginal and average profits:

$$\pi_{i,t}L_{i,t} - \kappa V_{i,t} = \alpha A_{i,t}L_{i,t}^{\alpha} - W_{i,t}L_{i,t} - \kappa V_{i,t} = E_{i,t} - (1-\alpha)Y_{i,t}$$

where $E_{i,t} \equiv \Pi_{i,t} - \kappa V_{i,t}$ is the firm's earnings, $\Pi_{i,t} \equiv Y_{i,t} - W_{i,t}L_{i,t} = A_{i,t}L_{i,t}^{\alpha} - W_{i,t}L_{i,t}$ is the total profit before wages $W_{i,t}L_{i,t}$ and vacancy posting costs $\kappa V_{i,t}$, and $\pi_{i,t} = \frac{\partial \Pi_{i,t}}{\partial L_{i,t}}$ is the marginal profit from hiring. The second term $(1-\alpha)Y_{i,t}$ is a "DRS wedge" that captures the gap between the average profit and marginal profit. Under DRS, the firm's hiring condition becomes:

$$\frac{\kappa}{q_t} = \mathbb{F}_t\left[\sum_{j=1}^{\infty} \frac{1}{R_{i,t,t+j}}\left(\frac{E_{i,t+j}}{L_{i,t+1}} - (1-\alpha)\frac{Y_{i,t+j}}{L_{i,t+1}}\right)\right]$$

where firm i takes the aggregate vacancy filling rate q_t as given. Express aggregate earning-employment and output-employment ratios as the employment-weighted average of firm-level ratios:

$$\frac{\kappa}{q_t} = \mathbb{F}_t\Big[\sum_i \sum_{j=1}^{\infty} \frac{1}{R_{i,t,t+j}}\Big(\frac{E_{i,t+j}}{L_{i,t+1}} - (1-\alpha)\frac{Y_{i,t+j}}{L_{i,t+1}}\Big)\frac{L_{i,t+1}}{L_{t+1}}\Big]$$

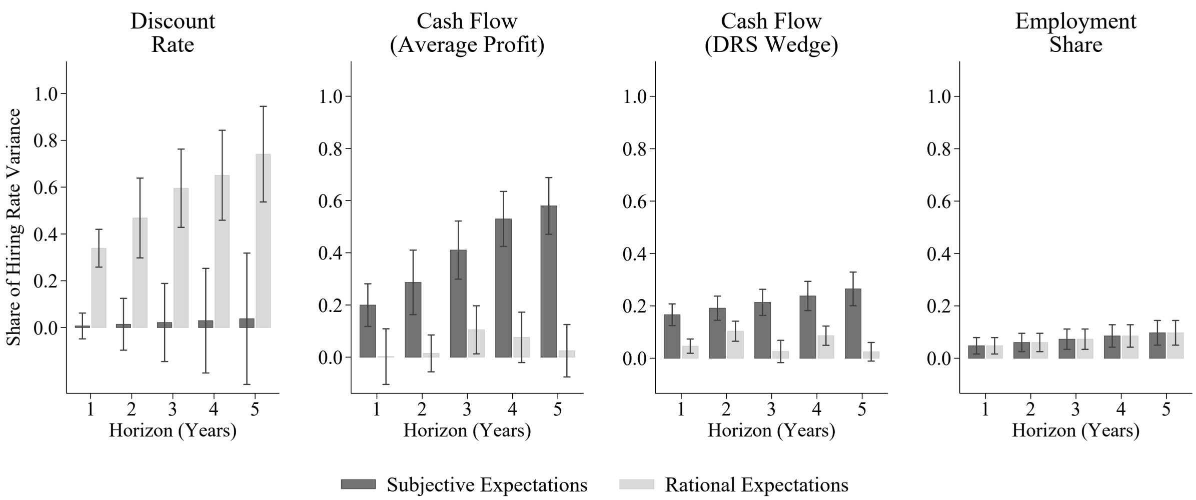
Define $S_{i,t+1} \equiv \frac{L_{i,t+1}}{L_{t+1}}$ as the employment share, $EL_{i,t+j} \equiv E_{i,t+j}/L_{i,t+1}$ the earnings-employment ratio, and $YL_{i,t+j} \equiv Y_{i,t+j}/L_{i,t+1}$ the output-employment ratio of firm i. Log linearize the expression around the steady state:

$$\log q_t = \sum_{j=1}^{\infty}\sum_i \Bigg[\underbrace{\mathbb{F}_t\left[w_{r,i,j}r_{i,t,t+j}\right]}_{\text{Discount Rate}} - \underbrace{\mathbb{F}_t\left[w_{el,i,j}el_{i,t+j}\right]}_{\substack{\text{Cash Flow}\\\text{(Earnings-Employment)}}} + \underbrace{\mathbb{F}_t\left[w_{yl,i,j}yl_{i,t+j}\right]}_{\substack{\text{Cash Flow}\\\text{(Output-Employment)}}} - \underbrace{\mathbb{F}_t\left[w_{s,i,j}s_{i,t+1}\right]}_{\text{Employment Share}}\Bigg] \tag{OA.9}$$

where $r_{i,t,t+j}$, $el_{i,t+j}$, $yl_{i,t+j}$, and $s_{i,t+1}$ denote log deviations of $R_{i,t,t+j}$, $EL_{i,t+j}$, $YL_{i,t+j}$, and $S_{i,t+1}$ from the steady state state, respectively. The coefficients $w_{r,i,j} = w_{s,i,j} \equiv \frac{\overline{q}}{\kappa}\frac{(\overline{EL}_i+(1-\alpha)\overline{YL}_i)\cdot\overline{S}_i}{\overline{R}_i^j}$, $w_{el,i,j} \equiv \frac{\overline{q}}{\kappa}\frac{\overline{EL}_i\cdot\overline{S}_i}{\overline{R}_i^j}$, and $w_{yl,i,j} \equiv (1-\alpha)\frac{\overline{q}}{\kappa}\frac{\overline{YL}_i\cdot\overline{S}_i}{\overline{R}_i^j}$ are functions of steady-state values and linearization constants. Note that $s_{i,t+1}$ is constant in j and that the effective weight is the sum of the $w_{s,i,j}$'s. $\alpha = 0.72$ comes from the labor share, $\kappa = 0.133$ comes from the flow vacancy cost (Elsby and Michaels, 2013). $\overline{q} = 0.631$, $\overline{R}_i = 1.04$, $\overline{EL} = 0.014$, $\overline{YL} = 0.074$ are long-run sample averages. Finally, approximate the infinite sum by truncating up to h periods.

The expected output-employment ratio $\mathbb{F}_t[yl_{i,t+j}]$ captures the DRS wedge, and the employment share $s_{i,t+1}$ captures composition effects of changes in the firm size distribution. I measure the expected output-employment ratio $\mathbb{F}_t[yl_{i,t+j}]$ by using IBES sales forecasts. Figure OA.11 shows that under subjective expectations, the output-employment term accounts for roughly 30% of the variation in the vacancy filling rate, while the earnings-employment term explains slightly less than 60%. The compositional term is small. These results confirm that even under DRS, subjective cash flow expectations, whether expressed in average or marginal terms, remain the dominant driver of hiring fluctuations.

Figure OA.11: Time-Series Decomposition of the Vacancy Filling Rate: Decreasing Returns to Scale



Notes: This figure illustrates the components of the time-series decomposition of aggregate vacancy filling rate under decreasing returns to scale, based on equation (OA.9). The components of the decomposition are expected present discounted values of discount rate, earnings-employment ratio, output-employment ratio, and the employment share. The light bars show the contributions to the vacancy filling rate obtained under objective expectations. The dark bars show the contributions to the time-series variation in the vacancy filling rate obtained in subjective expectations. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts of CFOs and IBES financial analysts. Objective expectations $\mathbb{E}_t$ are based on machine learning forecasts from Long Short-Term Memory (LSTM) neural networks. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4.

OA.5 Gradual Adjustment of Expectations

To provide evidence on the dynamics of belief formation, this section examines how survey respondents revise their expectations about future earnings following an earnings surprise. The following regression estimates the responsiveness of long-horizon forecasts to short-term earnings news:

$$\mathbb{F}_{t+j}[\widetilde{x}_{i,t+h}] - \mathbb{F}_{t+j-1}[\widetilde{x}_{i,t+h}] = \alpha_{h,j} + \gamma_{h,j}(\widetilde{x}_{i,t+1} - \mathbb{F}_t[\widetilde{x}_{i,t+1}]) + \eta_{h,t+j},$$

where $\mathbb{F}_{t+j}[\widetilde{x}_{i,t+h}]$ denotes the expectation formed at time $t+j$ for earnings-related variable $\widetilde{x}$ at horizon h, and $\widetilde{x}_{i,t+1} - \mathbb{F}_t[\widetilde{x}_{i,t+1}]$ captures the earnings surprise. The coefficient $\gamma_{h,j}$ measures how much of the surprise is incorporated into expectations for long-run outcomes.

Table OA.7 reports estimates for two forward-looking variables: (a) long-run earnings growth, and (b) the long-run ratio of earnings to employment. The target horizon is fixed at $h = 5$ years, while the revision horizon j ranges from 1 to 4 years. The estimated $\gamma_{h,j}$ coefficients are uniformly small and often statistically insignificant, indicating that respondents only partially incorporate short-term earnings news into their long-run expectations. This pattern is consistent with models of belief formation under constant-gain learning, in which agents update expectations gradually and exhibit fading memory. In such models, a fixed updating gain leads to persistent deviations from rational expectations and a breakdown of the law of iterated expectations.

Table OA.7: Gradual adjustment of expectations

Target Horizon h (Years)	5	5	5	5
Revision Horizon j (Years)	1	2	3	4
Survey Forecast Revisions: $\mathbb{F}_{t+j}[\widetilde{x}_{i,t+h}] - \mathbb{F}_{t+j-1}[\widetilde{x}_{i,t+h}] = \alpha_{h,j} + \gamma_{h,j}(\widetilde{x}_{i,t+1} - \mathbb{F}_t[\widetilde{x}_{i,t+1}]) + \eta_{h,t+j}$				
(a) Earnings Growth	0.0929	0.0934	0.1121	0.1245
	(0.0734)	(0.0455)	(0.0776)	(0.0743)
(b) Earnings to Employment	0.0600	0.0508	0.0697	0.0745
	(0.1281)	(0.0725)	(0.0321)	(0.0419)

Notes: Table shows the gradual adjustment of expectations about future earnings $\widetilde{x}_{i,t+h}$ after an earnings surprise at $t+1$. Sample: 2005Q1 to 2023Q4. Newey-West t-statistics with lags = 4 reported in parentheses: *sig. at 10%. **sig. at 5%. ***sig. at 1%.

OA.6 Subjective User Cost of Labor

Overview The previous sections show that firms' hiring decisions are heavily influenced by subjective cash flow expectations. This section examines whether expectations about the user cost of labor also contribute to hiring behavior, since it is a key component of the firm's cash flows. Using survey data, I show that subjective wage expectations are significantly less cyclical than realized wages, implying that firms perceive labor costs as more rigid than they actually are. To account for the possibility that wages depend on the economic conditions at the start of the job, I use survey expectations from the SCE to measure the user cost of labor under subjective expectations.

In the search and matching model, the user cost of labor is the difference in the expected present value of wages between two firm-worker matches that are formed in two consecutive periods. Existing work assumes full information rational expectations and show that this user cost is more cyclical than flow wages, as workers hired in recessions earn lower wages both when hired and over time (Kudlyak, 2014; Bils et al., 2023). This section relaxes that assumption by using survey-based measures of subjective wage expectations. If firms and workers perceive the future path of wages as rigid, the subjective user cost of labor may remain high even during recessions, amplifying hiring and unemployment fluctuations.

Wages To assess the cyclicality of subjective wage expectations, I use publicly available survey and macroeconomic data to construct measures of actual real wage growth, subjective wage expectations, and unemployment rate changes. The Livingston Survey (semi-annual, 1961S1-2022S2), the CFO Survey (quarterly, 2001Q4-2023Q4), and the Survey of Consumer Expectations (SCE) (monthly, 2015M5-2022M12) provide the necessary data. I derive subjective wage growth expectations from median consensus forecasts of nominal wage growth in these surveys. The Livingston Survey forecasts are deflated using its own median CPI inflation forecast, while the CFO and SCE survey forecasts are deflated using CPI inflation expectations from the Survey of Professional Forecasters (SPF).

To account for the possibility that wages depend on the economic conditions at the start of the job, I use survey expectations from the SCE to measure the user cost of labor UC_t^W under subjective expectations. In the search and matching model, the user cost of labor is the difference in the present value of expected wages between two firm-worker matches that are formed in two consecutive periods. I use survey data to directly measure the expected wages. Consider the free-entry condition in the search and matching model:

$$\frac{\kappa}{q_t} = J_{t,t}$$

where a firm must pay a per vacancy cost of κ and vacancies are filled with probability q_t. $J_{t,\tau}$ is the value of a firm with a worker at time τ such that the productive match started at time t:

$$J_{t,\tau} \equiv z_t - w_{t,t} + \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} \mathbb{F}_t[z_\tau - w_{t,\tau}]$$

where $\mathbb{F}_t[\cdot]$ denotes subjective expectations based on survey data. $\beta = 0.9569$ is a discount factor and $\delta = 0.295$ is the probability that an employment relationship is terminated, both from Kudlyak (2014). Each period τ, a firm-worker match produces a per period output of z_τ and an employed worker received wage $w_{t,\tau}$ where t is the period when the worker is hired. $w_{t,t}$ is the new-hire wage. Note that the free entry condition is only required to hold for newly created matches for $\tau = t$. The expected difference between the firm's value of a newly created match in time t and the discounted value of a newly created match in period $t+1$ is

$$\begin{aligned} J_{t,t} - \beta(1-\delta)\mathbb{F}_t[J_{t+1,t+1}] &= z_t - w_{t,t} + \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} \mathbb{F}_t[z_\tau - w_{t,\tau}] \\ &- \beta(1-\delta)\mathbb{F}_t\left[z_{t+1} - w_{t+1,t+1} + \sum_{\tau=t+2}^{\infty} (\beta(1-\delta))^{\tau-(t+1)} \mathbb{F}_{t+1}[z_\tau - w_{t+1,\tau}]\right] \end{aligned}$$

Assuming for simplicity that the Law of Iterated Expectations holds under subjective beliefs:

$$J_{t,t} - \beta(1-\delta)\mathbb{F}_t[J_{t+1,t+1}] = z_t - w_{t,t} - \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} \mathbb{F}_t[w_{t,\tau} - w_{t+1,\tau}]$$

Substitute the free-entry condition to the left-hand side

$$\underbrace{\frac{\kappa}{q_t} - \beta(1-\delta)\mathbb{F}_t\left[\frac{\kappa}{q_{t+1}}\right]}_{\text{Non-wage component of user cost } UC_t^V} = \underbrace{z_t}_{\text{Benefit}} - \underbrace{\left[w_{t,t} + \sum_{\tau=t+1}^{\infty} (\beta(1-\delta))^{\tau-t} \mathbb{F}_t[w_{t,\tau} - w_{t+1,\tau}]\right]}_{\text{Wage component of user cost } UC_t^W}$$

The equation shows that the firm faces two sources of costs from a match: wage payments to a worker UC_t^W and vacancy opening costs UC_t^V. The firm creates jobs as long as the marginal benefit from adding a worker exceeds the user cost of labor. Note that the wage component of the user cost of labor UC_t^W, not the wage $w_{t,t}$, is the allocative price of labor.

I use aggregate-level data from the Livingston and CFO Surveys, along with worker-level data from the Survey of Consumer Expectations (SCE), to construct the user cost of labor UC_t^W under the survey respondents' subjective expectations. The SCE asks respondents about: the *month and year on which their current employment relationship started* (i.e., t in $w_{t,\tau}$); *"annual earnings, before taxes and other deductions, on your [current/main] job"* ($w_{t,\tau}$); short-term expectations on what their *"annual earnings will be in 4 months"* ($\mathbb{F}_t[w_{t,t+\frac{4}{12}}]$) and long-term expectations on *"annual earnings to be at your current job in 10 years"* ($\mathbb{F}_t[w_{t,t+10}]$). I obtain survey expectations about medium-term earnings between 4 months to 10 years by linearly interpolating between the two horizons:

$$\mathbb{F}_t[w_{t,t+h}] = \frac{10-h}{10-\frac{4}{12}}\mathbb{F}_t[w_{t,t+\frac{4}{12}}] + \frac{h-\frac{4}{12}}{10-\frac{4}{12}}\mathbb{F}_t[w_{t,t+10}], \qquad h = 1, 2, \ldots, 10$$

The user cost of labor formulation assumes infinitely lived firms and workers, while empirical data are inherently finite. I truncate the horizon at 10 years given the availability of the survey data. Longer horizons reduce the weight of future terms due to discounting and job separations. In addition, if unemployment follows a mean-reverting process, wages in long-term employment relationships will eventually converge to the long-term mean, which after discounting would limit the size of very long-term influences (Kudlyak, 2014).

I measure actual real wage growth using two BLS wage series. The Livingston Survey forecasts target annual log real wage growth based on average weekly earnings of production and nonsupervisory employees in manufacturing (CES3000000030). The CFO and SCE surveys target annual log real wage growth based on average hourly earnings of private-sector employees (CEU0500000008). I deflate nominal wages using the Consumer Price Index (CPIAUCSL) to adjust for purchasing power.

For unemployment rates used to assess the cyclicality of wages, I use both actual data and subjective forecasts. Actual seasonally adjusted U.S. unemployment rate (UNRATE) comes from the BLS Current Population Survey (CPS). Subjective unemployment expectations are derived from median consensus SPF forecasts of future unemployment rates.

Time-series evidence Figure OA.12 compares realized real wage growth with 1-year-ahead subjective wage growth forecasts from three sources: the Livingston Survey, the CFO Survey, and the Survey of Consumer Expectations (SCE). Actual wage growth is clearly cyclical, with declines during downturns and strong rebounds during recoveries. In contrast, subjective wage forecasts are far more stable over time. Even during major shocks, such as the 2008 financial crisis and the COVID-19 recession, survey respondents anticipated only modest wage adjustments. Forecast errors are persistent and systematically biased: wage growth forecasts are excessively high during downturns and excessively low during expansions.

To formally assess the cyclicality of real wage growth, Table OA.8 panel (a) compares the relationship between changes in the unemployment rate and real wage growth across rational and subjective expectations. As a full information rational expectations (FIRE) benchmark, I use historical data on actual real wage growth to estimate the following regression, replicating existing estimates in the literature (e.g., Bils, 1985; Solon et al., 1994; Gertler et al., 2020):

$$\Delta \log w_t = \beta_0 + \beta_1 \Delta u_t + \varepsilon_t$$

where $\Delta \log w_t$ represents the actual annual log growth rate of real wages, Δu_t is the annual change in the unemployment rate, and ε_t is the error term. β_1 is the coefficient of interest and captures the cyclicality of real wage growth.

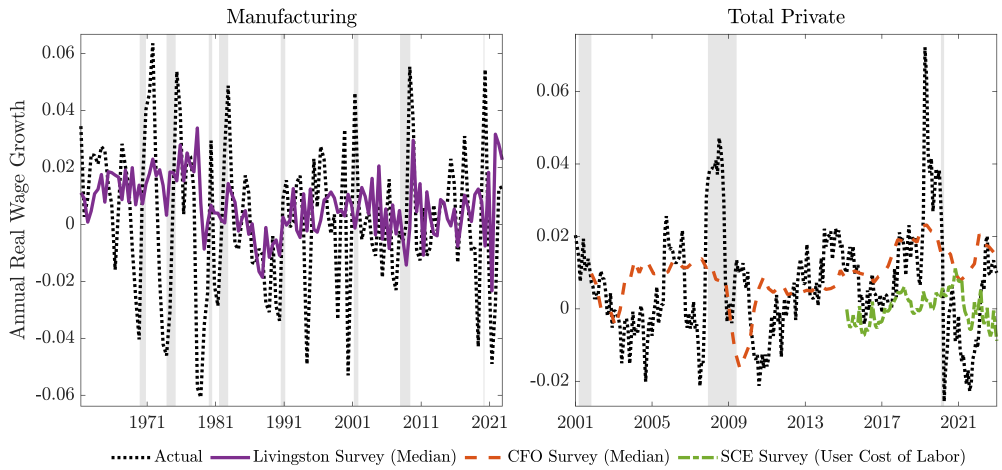
Under subjective expectations, I use survey data on expected real wage growth to estimate:

$$\mathbb{F}_{t-1}[\Delta \log w_t] = \beta_0 + \beta_1 \mathbb{F}_{t-1}[\Delta u_t] + \varepsilon_t$$

where $\mathbb{F}_{t-1}[\Delta \log w_t]$ is the median survey forecast for the annual log growth rate of real wages, where the surveys are either from Livingston, CFO, or SCE. $\mathbb{F}_{t-1}[\Delta u_t]$ is the median survey forecast of the annual change in the unemployment rate from the Survey of Professional Forecasters (SPF). The coefficient of interest β_1 measures the cyclicality of expected real wage growth as perceived by survey respondents.

Table OA.8 panel (a) reports the estimates. Under rational expectations, actual real wage growth is clearly cyclical since it is significantly negatively related to changes in unemployment rates. The magnitude of the estimate is also consistent with prior estimates in the literature, with elasticities ranging from -3.05 to -3.46 depending on the sample period (Solon et al., 1994). In contrast, subjective wage growth expectations are acyclical, with small and statistically insignificant coefficients across all survey sources and sample periods. Notably, the magnitude of the estimated elasticity is an order of magnitude smaller, ranging from -0.20 to -0.97 depending on the survey measure and sample period.

Figure OA.12: Real Wage Growth: Actual vs. Subjective Expectations



Notes: This figure plots ex-post realized outcomes (Actual) and 1-year ahead subjective expectations (Survey) of real wage growth. x axis denotes the date on which actual values were realized and the period on which the survey forecast is made, making the vertical distance between the actual and survey lines the forecast error. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts. Left panel compares actual values of annual log real wage growth against the median consensus forecasts from the Livingston survey, where wages are measured using average weekly earnings of production and nonsupervisory employees, manufacturing (CES3000000030). Right panel compares annual log real wage growth against median consensus forecasts from the CFO survey and the subjective user cost of labor measured from the Survey of Consumer Expectations (SCE), where wages are measured using average hourly earnings of production and nonsupervisory employees, total private (CEU0500000008). Actual values are deflated using the Consumer Price Index (CPIAUCSL). Livingston, CFO, and SCE survey expectations of nominal wage growth are deflated using median consensus forecasts of CPI inflation from the Livingston, SPF, and SCE surveys, respectively. The sample period for Livingston is semi-annual spanning 1961S1 to 2022S2, CFO survey is quarterly spanning 2001Q4 to 2023Q4, SCE is monthly spanning 2015M5 to 2022M12. NBER recessions are shown with gray shaded bars.

Cross-Sectional evidence To explore these patterns at the individual level, I use microdata from the SCE to estimate subjective wage cyclicality separately for new hires and incumbents. The regression specification relaxes the rational expectations assumption from Gertler et al. (2020) and includes an interaction between expected unemployment growth and the probability of being a new hire:

$$\mathbb{F}_{t-1}[\Delta \log w_{i,t}] = \beta_0 + \beta_1 \mathbb{F}_{t-1}[\Delta u_t] + \mathbb{F}_{t-1}[\mathbb{I}\{N_{i,t} = 1\}] \cdot [\beta_2 + \beta_3 \mathbb{F}_{t-1}[\Delta u_t]] + \varepsilon_{i,t}$$

where $\mathbb{F}_{t-1}[\Delta \log w_{i,t}]$ represents the time $t-1$ subjective expectation of wage growth for worker i at time t. $\mathbb{F}_{t-1}[\Delta u_t]$ is the survey expectation of aggregate unemployment growth. The indicator variable $\mathbb{I}\{N_{i,t} = 1\}$ equals one if the worker is newly hired and zero otherwise. Its expectation $\mathbb{F}_{t-1}[\mathbb{I}\{N_{i,t} = 1\}]$ is thus the subjective probability that the worker will be newly hired next period. The interaction term $\mathbb{F}_{t-1}[\mathbb{I}\{N_{i,t} = 1\}] \cdot \mathbb{F}_{t-1}[\Delta u_t]$ captures the differential sensitivity of expected wage growth to unemployment changes for new hires relative to incumbents. The error term $\varepsilon_{i,t}$ accounts for individual-level deviations in expectations. The coefficient β_1 captures the overall cyclicality of subjective wage expectations for workers whose new-hire probability is zero (incumbents), reflecting how much workers expect wages to change in response to shifts in aggregate unemployment. The coefficient β_2 measures the baseline difference in expected wage growth between new hires and existing workers when expected unemployment growth is zero. The interaction term β_3 determines whether new hires expect wages to be more sensitive to unemployment fluctuations than incumbents do.

The results in Table OA.8 panel (b) column (1) show that, even after controlling for differences between job stayers and new hires, subjective wage expectations are highly rigid and exhibit weak cyclicality. The coefficient β_1 is negative but small, confirming the aggregate result in panel (a) that workers that are not new hires expect only mild wage adjustments in response to unemployment fluctuations. The estimate for β_2 is positive, suggesting that, on average, new hires expect higher wage growth than job stayers. The interaction term β_3 is negative but small in magnitude, implying that new hires do not expect substantially greater cyclicality in wages compared to incumbents. Column (2) extends column (1) by including worker fixed effects to find similar results. These findings extend the results from aggregate regressions by showing that subjective wage expectations are highly rigid even at the individual level, regardless of job transitions. Both new hires and incumbents perceive only weak cyclical variation in wages.

Implications for macroeconomic models These findings could have important implications for macroeconomic models of unemployment fluctuations. If firms do not expect wages to fall during downturns, then the subjective user cost of labor remains high even as demand declines, suppressing job creation. This mechanism is consistent with models that rely

Table OA.8: Cyclicality of Real Wage Growth: Actual vs. Subjective Expectations

(a) Aggregate Time-Series
Actual: $\Delta \log w_t = \beta_0 + \beta_1 \Delta u_t + \varepsilon_t$
Subjective: $\mathbb{F}_{t-1}[\Delta \log w_t] = \beta_0 + \beta_1 \mathbb{F}_{t-1}[\Delta u_t] + \varepsilon_t$

	1961S1-2022S2		2001Q4-2023Q4		2015M5-2022M12	
	Actual	Survey Median (Liv)	Actual	Survey Median (CFO)	Actual	Survey User Cost (SCE)
	(1)	(2)	(3)	(4)	(5)	(6)
Unemployment Rate	-0.0340^{***}	-0.0020	-0.0305^{***}	0.0006	-0.0346^{***}	-0.0086
t-stat	(-3.8684)	(-0.1568)	(-4.2477)	(0.0800)	(-6.6994)	(-1.6332)
Adj. R^2	0.1021	0.0003	0.2557	0.0001	0.4719	0.0498
N	124	124	85	85	92	92
Frequency	SA	SA	Q	Q	M	M
Sector	Mfg	Mfg	Pvt	Pvt	Pvt	Pvt

(b) Worker-Level New Hire Effect
Subjective: $\mathbb{F}_{t-1}[\Delta \log w_{i,t}] = \beta_0 + \beta_1 \mathbb{F}_{t-1}[\Delta u_t] + \mathbb{F}_{t-1}[\mathbb{I}\{N_{i,t} = 1\}] \cdot [\beta_2 + \beta_3 \mathbb{F}_{t-1}[\Delta u_t]] + \varepsilon_{i,t}$

	2015M5-2022M12	
	Survey (SCE)	Survey (SCE)
	(1)	(2)
	First Difference	Fixed Effects
Unemployment Rate	-0.0048	-0.0028
	(0.0029)	(0.0026)
New Hire	0.0036^{***}	0.0003
	(0.0009)	(0.0013)
Unemployment Rate × New Hire	-0.0026	-0.0059
	(0.0020)	(0.0035)
Adj. R^2	0.0011	0.0036
N	39,832	39,832
Frequency	M	M
Sector	Pvt	Pvt

Notes: Table reports estimates from time-series and worker-level regressions of annual log real wage growth on unemployment growth. Subjective expectations $\mathbb{F}_t$ are based on survey forecasts. Panel (a) reports estimates from time-series regressions using the aggregate series. Panel (a) Columns (1)-(2) compare actual values of annual log real wage growth against the median consensus forecasts from the Livingston survey, where wages are measured using average weekly earnings of production and nonsupervisory employees, manufacturing (CES3000000030). Panel (a) Columns (3)-(6) compare compares annual log real wage growth against median consensus forecasts from the CFO survey and the subjective user cost of labor measured from the Survey of Consumer Expectations (SCE), where wages are measured using average hourly earnings of production and nonsupervisory employees, total private (CEU0500000008). Panel (b) reports worker-level estimates from regressions of SCE survey expectations of wage growth on survey expectations of unemployment growth, an indicator of whether the worker is a new hire, and the interaction between the two. Actual wage growth is deflated using the Consumer Price Index (CPIAUCSL). Livingston, CFO, and SCE survey expectations of nominal wage growth are deflated using median consensus forecasts of CPI inflation from the Livingston, SPF, and SCE surveys, respectively. Subjective expectations of unemployment rates are from 1-year ahead consensus median forecasts from the SPF. The sample period for Livingston is semi-annual spanning 1961S1 to 2022S2, CFO survey is quarterly spanning 2001Q4 to 2023Q4, SCE is monthly spanning 2015M5 to 2022M12. Panel (a): Newey-West corrected t-statistics with lags 2 (semi-annual), 4 (quarterly), 12 (monthly) are reported in parentheses; Panel (b): Standard errors clustered by worker are reported in parentheses. *sig. at 10%. **sig. at 5%. ***sig. at 1%.

on wage rigidity to explain labor market volatility (Shimer, 2005; Hall, 2005; Christiano et al., 2016). These results provide support for macroeconomists to introduce a rigid user cost of labor under subjective expectations to explain the volatility of business cycle fluctuations (e.g., Menzio (2023)). Moreover, the persistence of subjective wage expectations may reflect underlying frictions in information processing. Survey data on wage expectations can help distinguish between alternative theories of wage formation. Unlike rational models where the timing of wage payments is irrelevant (Barro, 1977), models with sticky or inattentive expectations, such as those in Mankiw and Reis (2002) or Coibion and Gorodnichenko (2015), can be better suited to capture the persistent behavior of expected wages. Finally, the finding that subjective cost of labor is rigid suggests that volatile subjective cash flow expectations are unlikely to be driven by fluctuations in the user cost of labor. Instead, firms are overreacting to other components of profitability, such as revenue expectations or perceived demand conditions, rather than expected changes in labor costs.

OB Model Details

OB.1 Representative Agent Model

In this section, I present a search and matching model based on Diamond (1982), Mortensen (1982), and Pissarides (2009). The model introduces subjective beliefs that may depart from rational expectations, thereby capturing the impact of belief distortions on labor market dynamics. See Petrosky-Nadeau et al. (2018) for a standard search and matching model formulated under rational expectations. I begin with a representative firm setup to develop intuition for the aggregate dynamics, then extend the model in Section OB.2 to include firm heterogeneity to support the cross-sectional analysis. Consider a discrete time economy populated by a representative household and a representative firm that uses labor as a single input to production.

Representative Household The household has a continuum of mass 1 members who are either employed L_t or unemployed U_t at any point in time. The population is normalized to 1, i.e., $L_t + U_t = 1$, meaning that L_t and U_t are also the rates of employment and unemployment, respectively. The household's consumption decision implies a stochastic discount factor M_{t+1}. The household pools the income of all members before making its consumption decision. Assume that the household has perfect consumption insurance and its members have access to complete contingent claims against aggregate risk. Risk sharing implies each member consumes the same amount regardless of idiosyncratic shocks.

Search and Matching At the start of period t, the employment stock L_t reflects the total number of workers carried over from the previous period before any separations or new hires in period t. A fraction δ_t of these workers separate during the period, so the number of continuing employees becomes $(1-\delta_t)L_t$. The representative firm posts job vacancies V_t and engages in search over the course of the period to attract unemployed workers U_t. Matches are formed at the end of period t according to a matching function $m(U_t, V_t)$, where $q_t \equiv m(U_t, V_t)/V_t$ is the *vacancy filling rate*, and $f_t \equiv m(U_t, V_t)/U_t$ is the *job finding rate*. These new matches become part of the workforce starting in period $t+1$, so employment evolves according to the employment accumulation equation:

$$L_{t+1} = (1-\delta_t)L_t + q_t V_t \tag{OA.10}$$

The vacancy filling rate q_t maps vacancy posting decisions made during period t into employment outcomes observed at the beginning of period $t+1$. The variance decomposition does not require us to fully specify the matching function m. Posting a vacancy costs the firm $\kappa > 0$ per period, reflecting fixed hiring costs such as training and administrative setup. Jobs are destroyed at a time-varying job separation rate δ_t. Unemployment $U_t = 1 - L_t$ evolves according to:

$$U_{t+1} = \delta_t(1-U_t) + (1-q_t\theta_t)U_t \tag{OA.11}$$

where $\theta_t = V_t/U_t$ denotes labor market tightness, defined as the vacancy-to-unemployment ratio.

Representative Firm The firm has access to a production function F which uses labor L_t as an input to produce output $Y_t = F(L_t)$. Dividends to the firm's shareholders E_t are defined as $E_t \equiv \Pi_t - \kappa V_t$, where $\Pi_t \equiv Y_t - W_t L_t$ is the total profit before vacancy posting costs κV_t and W_t is the wage rate. As in Petrosky-Nadeau et al. (2018), I assume that the representative household owns the equity of the firm, and that the firm pays out all of its earnings as dividends. I also assume that firms have the same unconstrained access to financing as investors in the financial market. The firm posts the optimal number of vacancies to maximize the cum-dividend market value of equity S_t:

$$S_t = \max_{\{V_{t+j}, L_{t+j}\}_{j=0}^{\infty}} \mathbb{F}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} E_{t+j} \right] \tag{OA.12}$$

subject to the employment accumulation equation (OA.10). The firm takes the wage rate W_t, household's stochastic discount factor $M_{t,t+j} = \prod_{s=1}^{j} M_{t+s}$, and vacancy filling rate q_t as given. $\mathbb{F}_t[\cdot]$ denotes expectations conditional on information available at period t, computed based on the firm's possibly distorted beliefs. These beliefs may depart from objective expectations $\mathbb{E}_t[\cdot]$, with the nature and magnitude of the deviation disciplined using survey data.

Hiring Equation The firm's optimal hiring decision equates the expected discounted value of hiring a marginal worker with its marginal cost. Rewrite the firm's problem in equation (OA.12) from infinite-horizon to recursive form:

$$S_t = \max_{V_t, L_{t+1}} \Pi_t - \kappa V_t + \mathbb{F}_t \left[M_{t+1} S_{t+1} \right] \tag{OA.13}$$

$$\text{s.t. } L_{t+1} = (1-\delta_t)L_t + q_t V_t \tag{OA.14}$$

The first-order condition with respect to V_t is:

$$\frac{\partial S_t}{\partial V_t} = -\kappa + \mathbb{F}_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial L_{t+1}}\frac{\partial L_{t+1}}{\partial V_t}\right] = 0 \tag{OA.15}$$

Substitute $\frac{\partial L_{t+1}}{\partial V_t} = q_t$ and $\frac{\partial L_{t+1}}{\partial L_t} = (1-\delta_t)$ from the employment accumulation equation (OA.14), and rearrange (OA.15) in terms of the marginal cost of hiring κ/q_t:

$$\frac{\kappa}{q_t} = \mathbb{F}_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial L_{t+1}}\right] \tag{OA.16}$$

Next, differentiate S_t with respect to L_t:

$$\frac{\partial S_t}{\partial L_t} = \frac{\partial \Pi_t}{\partial L_t} + \mathbb{F}_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial L_{t+1}}\frac{\partial L_{t+1}}{\partial L_t}\right] \tag{OA.17}$$

Substitute $\frac{\partial L_{t+1}}{\partial L_t} = (1-\delta_t)$ from the employment accumulation equation (OA.14):

$$\frac{\partial S_t}{\partial L_t} = \frac{\partial \Pi_t}{\partial L_t} + (1-\delta_t)\mathbb{F}_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial L_{t+1}}\right] \tag{OA.18}$$

Substitute equation (OA.18) for period $t+1$ into equation (OA.16):

$$\frac{\kappa}{q_t} = \mathbb{F}_t\left[M_{t+1}\left(\frac{\partial \Pi_{t+1}}{\partial L_{t+1}} + (1-\delta_{t+1})\mathbb{F}_{t+1}\left[M_{t+2}\frac{\partial S_{t+2}}{\partial L_{t+2}}\right]\right)\right] \tag{OA.19}$$

Finally, substitute in (OA.16) for period $t+1$ to arrive at the *hiring equation*:

$$\underbrace{\frac{\kappa}{q_t}}_{\text{Cost of hiring}} = \underbrace{\mathbb{F}_t\left[M_{t+1}\left(\pi_{t+1} + (1-\delta_{t+1})\frac{\kappa}{q_{t+1}}\right)\right]}_{\text{Expected discounted value of hiring}} \tag{OA.20}$$

where $\pi_t \equiv \frac{\partial \Pi_t}{\partial L_t}$ is the profit flow from the marginal hired worker. The hiring equation relates the marginal cost of hiring $\frac{\kappa}{q_t}$ with the expected marginal value of hiring to the firm, which equals the future expected marginal benefits of hiring discounted to present value with the stochastic discount factor M_{t+1}. The future marginal benefits of hiring include π_{t+1}, the future marginal product of labor net of the wage rate, plus the future marginal value of hiring, which equals the future marginal cost of hiring $\frac{\kappa}{q_{t+1}}$ net of separation $(1-\delta_{t+1})$. During recessions, vacancy filling rates q_t are high, which makes the cost of hiring κ/q_t low. The low cost of hiring must be rationalized by either low expected discounted profit flows $\mathbb{F}_t[M_{t+1}\pi_{t+1}]$ or low future value of hiring $(1-\delta_{t+1})\frac{\kappa}{q_{t+1}}$. The hiring equation is the labor market analogue of the optimality condition for physical capital in the q theory of investment (Hayashi, 1982), where κ/q_t is the upfront cost of investment analogous to Tobin's marginal q and δ_{t+1} is the depreciation rate.

Constant Returns to Scale (CRS) Next, I derive the firm's stock price implied by the optimal hiring decision. Assume a constant returns to scale (CRS) production function so that marginal profits equal average profits:

$$\pi_{t+1}L_{t+1} = \frac{\partial \Pi_{t+1}}{\partial L_{t+1}}L_{t+1} = \Pi_{t+1} \tag{OA.21}$$

Multiply both sides of the hiring equation by the number of employees L_{t+1}:

$$\frac{\kappa}{q_t}L_{t+1} = \mathbb{F}_t\left[M_{t+1}\left(\pi_{t+1}L_{t+1} + (1-\delta_{t+1})\frac{\kappa}{q_{t+1}}L_{t+1}\right)\right] \tag{OA.22}$$

Substitute in the employment accumulation equation (OA.14) and rearrange terms:

$$\frac{\kappa}{q_t}L_{t+1} = \mathbb{F}_t\left[M_{t+1}\left(\pi_{t+1}L_{t+1} + \frac{\kappa}{q_{t+1}}(L_{t+2} - q_{t+1}V_{t+1})\right)\right] \tag{OA.23}$$

$$= \mathbb{F}_t\left[M_{t+1}\left(\pi_{t+1}L_{t+1} - \kappa V_{t+1} + \frac{\kappa}{q_{t+1}}L_{t+2}\right)\right] \tag{OA.24}$$

Use the constant returns to scale assumption to simplify $\pi_{t+1}L_{t+1} - \kappa V_{t+1} = \Pi_{t+1} - \kappa V_{t+1} = E_{t+1}$:

$$\frac{\kappa}{q_t}L_{t+1} = \mathbb{F}_t\left[M_{t+1}\left(E_{t+1} + \frac{\kappa}{q_{t+1}}L_{t+2}\right)\right] \tag{OA.25}$$

Substitute the equation recursively:

$$\frac{\kappa}{q_t}L_{t+1} = \mathbb{F}_t\left[\sum_{j=1}^{\infty} M_{t,t+j}E_{t+j}\right] + \lim_{T\to\infty}\mathbb{F}_t\left[M_{t,t+T}\frac{\kappa}{q_{t+T}}L_{t+T+1}\right] \tag{OA.26}$$

The first term on the right-hand side is the firm's stock price $P_t \equiv S_t - E_t$, which is the firm's ex-dividend equity value. Take the second term to zero by applying a transversality condition to arrive at an equation that relates the total cost of hiring with the firm's stock price:

$$\frac{\kappa}{q_t}L_{t+1} = P_t \tag{OA.27}$$

where employment L_{t+1} is determined at the end of date t under the timing convention from equation (OA.10). Take logarithms of both sides of the firm's stock price equation (OA.27) and rearrange terms:

$$\log\kappa - \log q_t = \log\frac{P_t}{L_{t+1}} = \log\frac{P_t}{E_t} - \log\frac{E_t}{L_{t+1}} \equiv pe_t - el_t \tag{OA.28}$$

where I define $pe_t \equiv \log\frac{P_t}{E_t}$ and $el_t \equiv \log\frac{E_t}{L_{t+1}}$ for notational convenience.

Log-linear Approximation of Price-Earnings Ratio To express the price-earnings ratio pe_t in terms of forward-looking variables, start by log-linearizing the price-dividend ratio $pd_t = \log(P_t/D_t)$ around its long-term average $\overline{pd}$ (Campbell and Shiller, 1988):

$$pd_t = c_{pd} + \Delta d_{t+1} - r_{t+1} + \rho pd_{t+1} \tag{OA.29}$$

where c_{pd} is a linearization constant, $r_{t+1} \equiv \log(\frac{P_{t+1}+D_{t+1}}{P_t})$ is the log stock return (with dividends), and $\rho \equiv \exp(\overline{pd})/(1+\exp(\overline{pd})) = 0.98$ is a persistence parameter that arises from the log linearization. Rewrite the equation in terms of log price-earnings instead of log price-dividends by using the identity $pe_t = pd_t + de_t$, where de_t log payout ratio:

$$pe_t = c_{pd} + \Delta e_{t+1} - r_{t+1} + \rho pe_{t+1} + (1-\rho)de_{t+1} \tag{OA.30}$$

Since $1-\rho \approx 0$ and the payout ratio de_t is bounded, $(1-\rho)de_{t+1}$ can be approximated as a constant, i.e., $c_{pe} \approx c_{pd} + (1-\rho)de_{t+1}$ (De La O et al., 2024):

$$pe_t \approx c_{pe} + \Delta e_{t+1} - r_{t+1} + \rho pe_{t+1} \tag{OA.31}$$

Recursively substitute for the next h periods

$$pe_t = \sum_{j=1}^{h}\rho^{j-1}(c_{pe} + \Delta e_{t+j} - r_{t+j}) + \rho^h pe_{t+h} \tag{OA.32}$$

Decomposition of Vacancy Filling Rate Substitute the log-linearized price-earnings ratio in equation (OA.32) into the hiring equation in equation (OA.28):

$$\log q_t = \log\kappa - pe_t - el_t = \log\kappa - \left[\sum_{j=1}^{h}\rho^{j-1}(c_{pe} + \Delta e_{t+j} - r_{t+j}) + \rho^h pe_{t+h}\right] - el_t \tag{OA.33}$$

Rearrange and collect terms to obtain an ex-post decomposition of the vacancy filling rate:

$$\log q_t = c_q + \underbrace{\sum_{j=1}^{h}\rho^{j-1}r_{t+j}}_{r_{t,t+h}} - \underbrace{\left[el_t + \sum_{j=1}^{h}\rho^{j-1}\Delta e_{t+j}\right]}_{e_{t,t+h}} - \underbrace{\rho^h pe_{t+h}}_{pe_{t,t+h}} \tag{OA.34}$$

where $c_q \equiv \log \kappa - \frac{c_{pe}(1-\rho^h)}{1-\rho}$ is a constant. The equation decomposes the vacancy filling rate into future discount rates $r_{t,t+h} \equiv \sum_{j=1}^{h} \rho^{j-1} r_{t+j}$, cash flows $e_{t,t+h} \equiv el_t + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{t+j}$, and price-earnings $pe_{t,t+h} \equiv \rho^h pe_{t+h}$. The cash flow component consists of one period ahead log earnings-employment el_t, which captures news about current cash flow fluctuations, and $j = 1, \ldots, h$ period ahead log earnings growth Δe_{t+j}, which captures news about future cash flows. The earnings-employment ratio can be interpreted as a measure of the marginal product of labor under constant returns to scale (David et al., 2022). $pe_{t,t+h}$ is a terminal value that captures other long-term influences beyond h periods into the future not already captured in discount rates and cash flows. Since equation (OA.34) holds both ex-ante and ex-post, it can be evaluated under either subjective or objective expectations. The *subjective decomposition* replaces ex-post realizations of future outcomes with their subjective expectations:

$$\log q_t = c_q + \underbrace{\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[r_{t+j}]}_{\mathbb{F}_t[r_{t,t+h}]} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta e_{t+j}] \right]}_{\mathbb{F}_t[e_{t,t+h}]} - \underbrace{\rho^h \mathbb{F}_t[pe_{t+h}]}_{\mathbb{F}_t[pe_{t,t+h}]} \tag{OA.35}$$

Alternatively, the *objective decomposition* replaces ex-post realizations of future outcomes with their objective expectations:

$$\log q_t = c_q + \underbrace{\sum_{j=1}^{h} \rho^{j-1} \mathbb{E}_t[r_{t+j}]}_{\mathbb{E}_t[r_{t,t+h}]} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1} \mathbb{E}_t[\Delta e_{t+j}] \right]}_{\mathbb{E}_t[e_{t,t+h}]} - \underbrace{\rho^h \mathbb{E}_t[pe_{t+h}]}_{\mathbb{E}_t[pe_{t,t+h}]} \tag{OA.36}$$

Comparing these decompositions can quantify how belief distortions affect the vacancy filling rate.

Estimation The econometrician can estimate the variance decomposition using predictive regressions of each expected outcome on the current vacancy filling rate. For the subjective decomposition, demean each variable in equation (OA.35), multiply both sides by the current log vacancy filling rate $\log q_t$, and take the sample average:

$$Var\left[\log q_t\right] = Cov\left[\mathbb{F}_t[r_{t,t+h}], \log q_t\right] - Cov\left[\mathbb{F}_t[e_{t,t+h}], \log q_t\right] - Cov\left[\mathbb{F}_t[pe_{t,t+h}], \log q_t\right] \tag{OA.37}$$

where $Var[\cdot]$ and $Cov[\cdot]$ are sample variances and covariances based on data observed over a historical sample. Finally, divide both sides by $Var\left[\log q_t\right]$ to decompose its variance:

$$1 = \underbrace{\frac{Cov\left[\mathbb{F}_t[r_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Discount Rate News}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[e_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Cash Flow News}} - \underbrace{\frac{Cov\left[\mathbb{F}_t[pe_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Future Price-Earnings News}} \tag{OA.38}$$

The left-hand side represents the full variability in vacancy filling rates, hence is equal to one. Each term on the right reflects the share explained by subjective expectations of discount rates, cash flows, or price-earnings ratios. Under stationarity, the econometrician can estimate these shares using the OLS coefficients from regressing $\mathbb{F}_t[r_{t,t+h}]$, $\mathbb{F}_t[e_{t,t+h}]$, and $\mathbb{F}_t[pe_{t,t+h}]$ on the current log vacancy filling rate $\log q_t$, respectively. Finally, the decomposition under objective expectations can be estimated similarly based on equation (OA.36) by replacing the subjective expectation $\mathbb{F}_t[\cdot]$ with its objective counterpart $\mathbb{E}_t[\cdot]$:

$$1 = \underbrace{\frac{Cov\left[\mathbb{E}_t[r_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Discount Rate News}} - \underbrace{\frac{Cov\left[\mathbb{E}_t[e_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Cash Flow News}} - \underbrace{\frac{Cov\left[\mathbb{E}_t[pe_{t,t+h}], \log q_t\right]}{Var\left[\log q_t\right]}}_{\text{Future Price-Earnings News}} \tag{OA.39}$$

Under stationarity, the econometrician can estimate these shares using the OLS coefficients from regressing $\mathbb{E}_t[r_{t,t+h}]$, $\mathbb{E}_t[e_{t,t+h}]$, and $\mathbb{E}_t[pe_{t,t+h}]$ on the current log vacancy filling rate $\log q_t$, respectively.

OB.2 Cross-Sectional Decomposition of Hiring Rate

The cross-sectional analysis employs a firm-level hiring framework that is the direct analogue of the aggregate representative firm search model. Both approaches derive from the same fundamental principle: firms hire until the marginal cost of hiring equals the marginal value of an additional worker. The key difference lies in the level of aggregation and the specific frictions that generate hiring costs. In the aggregate search model, linear vacancy posting costs (κ per vacancy) combined with constant returns to scale imply that marginal value equals average value, leading to the simplified hiring condition $\frac{\kappa}{q_t} = \frac{P_t}{L_{t+1}}$. For cross-sectional analysis, I retain firm-level heterogeneity and introduce convex adjustment costs that generate dispersion in hiring rates while preserving the core economic mechanism linking firm valuations to hiring decisions.

Consider firm i with production function:

$$Y_{i,t} = A_{i,t} L_{i,t}^{\alpha} \tag{OA.40}$$

where $A_{i,t}$ represents productivity and $L_{i,t}$ is labor input. The firm's earnings, net of hiring costs and wages, are:

$$E_{i,t} = Y_{i,t} - \phi\left(\frac{H_{i,t}}{L_{i,t}}\right) L_{i,t} - W_{i,t} L_{i,t} \tag{OA.41}$$

where $\phi(\cdot)$ captures convex adjustment costs for hiring at rate $H_{i,t}/L_{i,t}$, and $W_{i,t}$ is the equilibrium wage. The adjustment cost function $\phi(\cdot)$ represents the firm-level analogue of the aggregate matching friction. While the search model features linear vacancy costs that aggregate to determine the market-wide vacancy filling rate, individual firms face convex costs when rapidly adjusting their workforce due to capacity constraints in recruitment, training bottlenecks, and organizational frictions. Firm value satisfies the Bellman equation:

$$V(A_{i,t}, L_{i,t}) = \max_{H_{i,t}} \left\{ E_{i,t} + \mathbb{F}_t \left[\frac{M_{t+1}}{M_t} V(A_{i,t+1}, L_{i,t+1}) \right] \right\} \tag{OA.42}$$

subject to the employment accumulation equation:

$$L_{i,t+1} = (1 - \delta_{i,t}) L_{i,t} + H_{i,t} \tag{OA.43}$$

where M_t is the stochastic discount factor and $\delta_{i,t}$ is the job separation rate. The first-order condition with respect to hiring equates marginal cost to marginal benefit:

$$\phi'\left(\frac{H_{i,t}}{L_{i,t}}\right) = \mathbb{F}_t \left[\frac{M_{t+1}}{M_t} \frac{\partial V(A_{i,t+1}, L_{i,t+1})}{\partial L_{i,t+1}} \right] \tag{OA.44}$$

Under constant returns to scale, the envelope theorem yields $\frac{\partial V}{\partial L} = \frac{V}{L}$, allowing us to express the marginal value in terms of observable quantities:

$$\phi'\left(\frac{H_{i,t}}{L_{i,t}}\right) = \mathbb{F}_t \left[\frac{M_{t+1}}{M_t} \frac{V(A_{i,t+1}, L_{i,t+1})}{L_{i,t+1}} \right] = \frac{P_{i,t}}{L_{i,t+1}} \tag{OA.45}$$

where $P_{i,t}$ is the ex-dividend firm value (stock price). This hiring condition is the firm-level equivalent of the aggregate search model's condition $\frac{\kappa}{q_t} = \frac{P_t}{L_{t+1}}$. Both express the fundamental insight that hiring depends on the ratio of firm value to employment, but the cross-sectional version allows for firm-specific variation in both the adjustment cost parameters and the value-to-employment ratios. Assuming quadratic adjustment costs $\phi\left(\frac{H_{i,t}}{L_{i,t}}\right) = \frac{c_l}{2}\left(\frac{H_{i,t}}{L_{i,t}}\right)^2$, the marginal cost becomes $\phi'\left(\frac{H_{i,t}}{L_{i,t}}\right) = c_l \frac{H_{i,t}}{L_{i,t}}$, yielding:

$$c_l \frac{H_{i,t}}{L_{i,t}} = \frac{P_{i,t}}{L_{i,t+1}} \tag{OA.46}$$

Taking logs and decomposing the price-to-employment ratio:

$$\ln(c_l) + \ln\left(\frac{H_{i,t}}{L_{i,t}}\right) = \ln\left(\frac{P_{i,t}}{E_{i,t}}\right) - \ln\left(\frac{E_{i,t+1}}{E_{i,t}}\right) + \ln\left(\frac{E_{i,t+1}}{L_{i,t+1}}\right) \tag{OA.47}$$

Using lowercase letters to denote log variables, this becomes:

$$\ln(c_l) + hl_{i,t} = pe_{i,t} - \Delta e_{i,t+1} + el_{i,t+1} \tag{OA.48}$$

Substituting the Campbell-Shiller decomposition of the log price-earnings ratio:

$$pe_{i,t} \approx c + \sum_{j=1}^{h} \rho^{j-1} \Delta e_{i,t+j} - \sum_{j=1}^{h} \rho^{j-1} r_{i,t+j} + \rho^h pe_{i,t+h} \tag{OA.49}$$

Taking subjective expectations and cross-sectionally demeaning to eliminate common terms yields the final decomposition:

$$\widetilde{hl}_{i,t} \approx \underbrace{\mathbb{F}_t[\widetilde{el}_{i,t+1}] + \sum_{j=2}^{h} \rho^{j-1} \mathbb{F}_t[\Delta \widetilde{e}_{i,t+j}]}_{\text{Cash Flow}} \underbrace{- \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\widetilde{r}_{i,t+j}]}_{\text{Discount Rate}} \underbrace{+ \rho^h \mathbb{F}_t[\widetilde{pe}_{i,t+h}]}_{\text{Future Price-Earnings}} \tag{OA.50}$$

where $\widetilde{x}_{i,t}$ denotes the cross-sectionally demeaned variable. The variance decomposition follows directly from the hiring rate decomposition:

$$1 \approx \underbrace{\frac{\text{Cov}(\mathbb{F}_t[\widetilde{el}_{i,t,t+h}], \widetilde{hl}_{i,t})}{\text{Var}(\widetilde{hl}_{i,t})}}_{\text{CF}_h} + \underbrace{\frac{\text{Cov}(-\mathbb{F}_t[\widetilde{r}_{i,t,t+h}], \widetilde{hl}_{i,t})}{\text{Var}(\widetilde{hl}_{i,t})}}_{\text{DR}_h} + \underbrace{\frac{\text{Cov}(\mathbb{F}_t[\widetilde{pe}_{i,t,t+h}], \widetilde{hl}_{i,t})}{\text{Var}(\widetilde{hl}_{i,t})}}_{\text{PE}_h} \tag{OA.51}$$

where $\widetilde{el}_{i,t,t+h} \equiv el_{i,t+1} + \sum_{j=2}^{h} \rho^{j-1} \Delta\widetilde{e}_{i,t+j}$, $\widetilde{r}_{i,t,t+h} \equiv \sum_{j=1}^{h} \rho^{j-1} \widetilde{r}_{i,t+j}$, and $\widetilde{pe}_{i,t,t+h} \equiv \rho^h \widetilde{pe}_{i,t+h}$. These covariance terms are estimated as coefficients from univariate regressions with time fixed effects, allowing us to isolate the cross-sectional variation attributable to each component while controlling for aggregate time-series effects.

The data uses Compustat annual employment data ($L_{i,t}$, variable EMP) from 2000 to 2023. The firm-level hiring rate is constructed from the employment accumulation equation:

$$\frac{H_{i,t}}{L_{i,t}} = \frac{L_{i,t+1}}{L_{i,t}} - (1 - \delta_{i,t}) \tag{OA.52}$$

where the job separation rate $\delta_{i,t}$ uses industry-level data from JOLTS.

OB.3 Model of Constant-Gain Learning from Cash Flows

In this appendix, I provide the full technical details and derivations for the constant-gain learning model presented in Section 7. The model embeds belief distortions in a search-and-matching framework, showing how firms' subjective expectations about cash flows shape their vacancy posting decisions and drive variation in hiring and vacancy filling rates.

OB.3.1 Environment and Firm Problem

The model features a frictional labor market in which unemployed workers are matched with job vacancies using a Cobb-Douglas matching function:

$$\mathcal{M}(U_t, V_t) = B U_t^{\eta} V_t^{1-\eta} \tag{OA.53}$$

where $\mathcal{M}(U_t, V_t)$ denotes the total number of matches in period t and is a function of aggregate unemployment U_t and job vacancies V_t. B is the matching efficiency parameter, and $\eta \in (0, 1)$ governs the elasticity of matches with respect to unemployment. The probability that a firm fills a posted vacancy, the vacancy filling rate, is then given by:

$$q_t = \frac{\mathcal{M}(U_t, V_t)}{V_t} = B \left(\frac{U_t}{V_t}\right)^{\eta} = B\theta_t^{-\eta} \tag{OA.54}$$

where $\theta_t \equiv V_t/U_t$ denotes labor market tightness. A firm that posts a vacancy incurs a cost $\kappa > 0$ per period. Matches dissolve at an exogenous separation rate δ, and each firm hires new workers by posting vacancies in anticipation of future returns. Each firm i uses labor to produce output via a constant returns to scale (CRS) production function:

$$Y_{i,t} = A_{i,t} L_{i,t} \tag{OA.55}$$

where $A_{i,t}$ is firm-level productivity and $L_{i,t}$ is the level of employment. The firm pays wages $W_{i,t}$, incurs hiring costs $\kappa V_{i,t}$, and generates earnings:

$$E_{i,t} = Y_{i,t} - W_{i,t} L_{i,t} - \kappa V_{i,t} \tag{OA.56}$$

Earnings represent the net flow profits from operating the firm: output net of the wage bill and the costs associated with posting vacancies. Firms maximize the expected present discounted value of earnings. Let $\mathcal{V}(A_{i,t}, L_{i,t})$ denote the value of the firm as a function of current productivity and employment. The Bellman equation for the firm's dynamic problem is:

$$\mathcal{V}(A_{i,t}, L_{i,t}) = \max_{V_{i,t}, L_{i,t+1}} \{E_{i,t} + \mathbb{F}_t [M_{t+1} \mathcal{V}(A_{i,t+1}, L_{i,t+1})]\} \tag{OA.57}$$

The firm chooses the number of vacancies $V_{i,t}$ to post and the resulting employment $L_{i,t+1}$ to maximize the sum of current earnings and the discounted continuation value, formed under subjective expectations $\mathbb{F}_t[\cdot]$ and a stochastic discount factor M_{t+1}. Employment evolves according to the accumulation equation:

$$L_{i,t+1} = (1 - \delta) L_{i,t} + q_t V_{i,t} \tag{OA.58}$$

which states that next period's employment depends on retained workers $(1-\delta)L_{i,t}$ and new hires $q_t V_{i,t}$ from current vacancies. Under constant returns to scale, the firm's marginal value of labor equals average value, and the first-order condition with respect to $V_{i,t}$ simplifies to:

$$\frac{\kappa}{q_t} = \mathbb{F}_t\left[M_{t+1}\frac{\partial\mathcal{V}(A_{i,t+1},L_{i,t+1})}{\partial L_{i,t+1}}\right] = \frac{\mathbb{F}_t\left[M_{t+1}\mathcal{V}(A_{i,t+1},L_{i,t+1})\right]}{L_{i,t+1}} \equiv \frac{P_{i,t}}{L_{i,t+1}} \tag{OA.59}$$

This condition equates the marginal cost of hiring a worker today, κ/q_t, to the expected marginal benefit of that hire, defined as the expected continuation value per worker. The term $P_{i,t} \equiv \mathbb{F}_t\left[M_{t+1}\mathcal{V}(A_{i,t+1},L_{i,t+1})\right]$ denotes the firm's ex-dividend market value. Rewriting in logs:

$$\log q_t = \log\kappa - \log\left(\frac{P_{i,t}}{L_{i,t+1}}\right) = \log\kappa - \underbrace{\log\left(\frac{P_{i,t}}{E_{i,t}}\right)}_{\equiv pe_{i,t}} - \underbrace{\log\left(\frac{E_{i,t}}{L_{i,t+1}}\right)}_{\equiv el_{i,t}} \tag{OA.60}$$

where $pe_{i,t} \equiv \log(P_{i,t}/E_{i,t})$ is the log price-earnings ratio and $el_{i,t} \equiv \log(E_{i,t}/L_{i,t+1})$ is the log earnings per worker.

OB.3.2 Cash Flow Process

Firms do not have full knowledge of the stochastic processes governing their cash flows. Instead, they form beliefs about their long-run mean using constant-gain learning. Assume that the firm's cash flow process consists of aggregate and idiosyncratic components. Firm i's earnings at time t are given by:

$$E_{i,t} = E_t \cdot \widetilde{E}_{i,t} = \exp(e_t + \widetilde{e}_{i,t}) \tag{OA.61}$$

The aggregate component follows an AR(1) process:

$$e_t = \mu + \phi e_{t-1} + u_t, \quad u_t \sim \mathcal{N}(0,\sigma_u^2) \tag{OA.62}$$

where μ is the unknown long-run mean, $\phi < 1$ is the known persistence parameter, and u_t is an i.i.d. Gaussian innovation. The idiosyncratic component also follows an AR(1) process:

$$\widetilde{e}_{i,t} = \widetilde{\mu}_i + \widetilde{\phi}\widetilde{e}_{i,t-1} + v_{i,t}, \quad v_{i,t} \sim \mathcal{N}(0,\sigma_v^2) \tag{OA.63}$$

where $\widetilde{\mu}_i$ is a firm-specific long-run mean in earnings (unknown to the firm), $\widetilde{\phi} < 1$ is a known persistence parameter, and $v_{i,t}$ is an i.i.d. idiosyncratic shock.

OB.3.3 Subjective Expectations Under Constant-Gain Learning

Objectively, mean growth is identical across firms: $\mu = \widetilde{\mu}_i = 0$. Under subjective beliefs, however, agents do not observe the true long-run mean μ and $\widetilde{\mu}_i$. They employ constant-gain learning with updating rules:

$$\mathbb{F}_t[\mu] = \mathbb{F}_{t-1}[\mu] + \nu\left(\Delta e_t - \mathbb{F}_{t-1}[\Delta e_t]\right) \tag{OA.64}$$
$$\mathbb{F}_t[\widetilde{\mu}_i] = \mathbb{F}_{t-1}[\widetilde{\mu}_i] + \nu\left(\Delta\widetilde{e}_{i,t} - \mathbb{F}_{t-1}[\Delta\widetilde{e}_{i,t}]\right) \tag{OA.65}$$

where ν is the constant gain parameter governing the speed of learning. Starting with equation (OA.64), substitute in the assumed true cash flow growth $\Delta e_t = e_t - e_{t-1} = \mu + \phi e_{t-1} + u_t - e_{t-1} = \mu + (\phi-1)e_{t-1} + u_t$ to the learning rule:

$$\begin{aligned}
\mathbb{F}_t[\mu] &= \mathbb{F}_{t-1}[\mu] + \nu\left(\Delta e_t - \mathbb{F}_{t-1}[\Delta e_t]\right) \\
&= \mathbb{F}_{t-1}[\mu] + \nu\left((\mu + (\phi-1)e_{t-1} + u_t) - (\mathbb{F}_{t-1}[\mu] + (\phi-1)e_{t-1})\right) \\
&= (1-\nu)\mathbb{F}_{t-1}[\mu] + \nu(\mu + u_t)
\end{aligned} \tag{OA.66}$$

Similarly, for the idiosyncratic component, substitute $\Delta\widetilde{e}_{i,t} = \widetilde{\mu}_i + (\widetilde{\phi}-1)\widetilde{e}_{i,t-1} + v_{i,t}$ to the learning rule:

$$\begin{aligned}
\mathbb{F}_t[\widetilde{\mu}_i] &= \mathbb{F}_{t-1}[\widetilde{\mu}_i] + \nu\left(\Delta\widetilde{e}_{i,t} - \mathbb{F}_{t-1}[\Delta\widetilde{e}_{i,t}]\right) \\
&= \mathbb{F}_{t-1}[\widetilde{\mu}_i] + \nu\Big((\widetilde{\mu}_i + (\widetilde{\phi}-1)\widetilde{e}_{i,t-1} + v_{i,t}) - (\mathbb{F}_{t-1}[\widetilde{\mu}_i] + (\widetilde{\phi}-1)\widetilde{e}_{i,t-1})\Big) \\
&= (1-\nu)\mathbb{F}_{t-1}[\widetilde{\mu}_i] + \nu(\widetilde{\mu}_i + v_{i,t})
\end{aligned} \tag{OA.67}$$

These updating rules show that beliefs evolve as a weighted average of the previous belief and the true parameter plus the current shock, with weight ν on the new information.

In this simplified framework, I abstract from wage determination and workers' beliefs to isolate the role of firms' expectations. Wages and worker-side beliefs are thus treated as residual objects consistent with the assumed cash flow process. The cash flow dynamics themselves are disciplined using data on firms' realized and expected earnings, allowing the model to capture belief-driven fluctuations in hiring without imposing additional structure on wage setting or worker expectations. This simplification highlights that firms' belief distortions alone can generate large fluctuations in vacancy creation and employment. In a richer model, if workers' beliefs differ from firms' beliefs, such disagreement could introduce further frictions in wage bargaining and amplify the effects of belief distortions on labor market dynamics.

OB.3.4 Cash Flow Growth Expectations

Aggregate Earnings Growth Given the learning rule, firms forecast future earnings. For one period ahead:

$$\mathbb{F}_t[e_{t+1}] = \mathbb{F}_t[\mu + \phi e_t + u_{t+1}] = \mathbb{F}_t[\mu] + \phi e_t \tag{OA.68}$$

For arbitrary horizon h, iterate forward:

$$\begin{aligned}\mathbb{F}_t[e_{t+h}] &= \mathbb{F}_t[\mu + \phi e_{t+h-1} + u_{t+h}] \\ &= \mathbb{F}_t[\mu] + \phi(\mathbb{F}_t[\mu] + \phi\mathbb{F}_t[e_{t+h-2}]) \\ &= \mathbb{F}_t[\mu] \cdot \frac{1-\phi^h}{1-\phi} + \phi^h e_t\end{aligned} \tag{OA.69}$$

Therefore, expected earnings growth h periods ahead is:

$$\begin{aligned}\mathbb{F}_t[\Delta e_{t+h}] &= \mathbb{F}_t[e_{t+h}] - \mathbb{F}_t[e_{t+h-1}] \\ &= \mathbb{F}_t[\mu] \cdot \frac{\phi^{h-1} - \phi^h}{1-\phi} + e_t(\phi^h - \phi^{h-1}) \\ &= \phi^{h-1}\left(\mathbb{F}_t[\mu] + (\phi - 1)e_t\right)\end{aligned} \tag{OA.70}$$

Idiosyncratic Earnings Growth By analogous reasoning, for the idiosyncratic component:

$$\begin{aligned}\mathbb{F}_t[\widetilde{e}_{i,t+h}] &= \sum_{j=0}^{h-1} \widetilde{\phi}^j \mathbb{F}_t[\widetilde{\mu}_i] + \widetilde{\phi}^h \widetilde{e}_{i,t} \\ &= \mathbb{F}_t[\widetilde{\mu}_i] \cdot \frac{1-\widetilde{\phi}^h}{1-\widetilde{\phi}} + \widetilde{\phi}^h \widetilde{e}_{i,t}\end{aligned} \tag{OA.71}$$

And therefore:

$$\begin{aligned}\mathbb{F}_t[\Delta\widetilde{e}_{i,t+h}] &= \mathbb{F}_t[\widetilde{e}_{i,t+h}] - \mathbb{F}_t[\widetilde{e}_{i,t+h-1}] \\ &= \widetilde{\phi}^{h-1}\left(\mathbb{F}_t[\widetilde{\mu}_i] + (\widetilde{\phi} - 1)\widetilde{e}_{i,t}\right)\end{aligned} \tag{OA.72}$$

The forecast of firm-level earnings growth is:

$$\mathbb{F}_t[\Delta e_{i,t+h}] = \mathbb{F}_t[\Delta e_{t+h}] + \mathbb{F}_t[\Delta\widetilde{e}_{i,t+h}] \tag{OA.73}$$

OB.3.5 Aggregate Strip Prices: Full Derivation

The log stochastic discount factor is:

$$m_{t+1} = -r_f - \frac{1}{2}\gamma^2\sigma_u^2 - \gamma u_{t+1} \tag{OA.74}$$

where r_f is the risk-free rate, γ is the coefficient of relative risk aversion, and u_{t+1} is the aggregate shock. In levels:

$$M_{t+1} = \exp\{m_{t+1}\} = \exp\left\{-r_f - \frac{1}{2}\gamma^2\sigma_u^2 - \gamma u_{t+1}\right\} \tag{OA.75}$$

Let $P_t^{(h)}$ denote the time t price for an aggregate strip with a payoff equal to the aggregate component of firms' cash flows received h periods in the future. I conjecture a log-linear solution:

$$P_t^{(h)} = \exp\left\{A^{(h)} + B^{(h)}\mathbb{F}_t[\mu] + \phi^h e_t\right\} \tag{OA.76}$$

Verification by Backward Recursion The strip price must satisfy the recursive pricing equation:

$$\begin{aligned} P_t^{(h)} &= \mathbb{F}_t[M_{t+1}P_{t+1}^{(h-1)}] \\ &= \mathbb{F}_t\left[\exp\left\{-r_f - \frac{1}{2}\gamma^2\sigma_u^2 - \gamma u_{t+1} + A^{(h-1)} + B^{(h-1)}\mathbb{F}_{t+1}[\mu] + \phi^{h-1}e_{t+1}\right\}\right] \end{aligned} \tag{OA.77}$$

Substitute the learning rule $\mathbb{F}_{t+1}[\mu] = (1-\nu)\mathbb{F}_t[\mu] + \nu(\mu + u_{t+1})$ and the AR(1) process $e_{t+1} = \mu + \phi e_t + u_{t+1}$:

$$P_t^{(h)} = \mathbb{F}_t\left[\exp\left\{ -r_f - \frac{1}{2}\gamma^2\sigma_u^2 - \gamma u_{t+1} + A^{(h-1)} + B^{(h-1)}((1-\nu)\mathbb{F}_t[\mu] + \nu(\mu + u_{t+1})) + \phi^{h-1}(\mu + \phi e_t + u_{t+1})\right\}\right]$$

Collect terms not involving u_{t+1} outside the expectation, and use $\mathbb{F}_t[\exp\{cu_{t+1}\}] = \exp\{\frac{1}{2}c^2\sigma_u^2\}$ for constant c:

$$\begin{aligned} P_t^{(h)} = \exp\Bigg\{ &-r_f - \frac{1}{2}\gamma^2\sigma_u^2 + A^{(h-1)} + B^{(h-1)}(1-\nu)\mathbb{F}_t[\mu] \\ &+ (\nu B^{(h-1)} + \phi^{h-1})\mu + \phi^h e_t + \frac{1}{2}(\nu B^{(h-1)} + \phi^{h-1} - \gamma)^2\sigma_u^2\Bigg\} \\ &\times \mathbb{F}_t[\exp\{(\nu B^{(h-1)} + \phi^{h-1})\mu\}] \end{aligned} \tag{OA.78}$$

Since agents do not know μ, they form expectations over it. Under the belief that μ follows the updating rule, the expected value of $\exp\{(\nu B^{(h-1)} + \phi^{h-1})\mu\}$ involves the variance of the belief error. For small learning gains ν, the variance of $\mu - \mathbb{F}_t[\mu]$ is approximately $\nu^2\sigma_u^2/(2\nu - \nu^2) \approx \nu\sigma_u^2$:

$$\mathbb{F}_t[\exp\{(\nu B^{(h-1)} + \phi^{h-1})\mu\}] \approx \exp\{(\nu B^{(h-1)} + \phi^{h-1})\mathbb{F}_t[\mu] + \frac{1}{2}(\nu B^{(h-1)} + \phi^{h-1})^2\nu^2\sigma_u^2\} \tag{OA.79}$$

Combining all terms:

$$\begin{aligned} P_t^{(h)} = \exp\Bigg\{ &-r_f + A^{(h-1)} + (B^{(h-1)}(1-\nu) + \nu B^{(h-1)} + \phi^{h-1})\mathbb{F}_t[\mu] + \phi^h e_t \\ &-\frac{1}{2}\gamma^2\sigma_u^2 + \frac{1}{2}(\nu B^{(h-1)} + \phi^{h-1} - \gamma)^2\sigma_u^2 + \frac{1}{2}(\nu B^{(h-1)} + \phi^{h-1})^2\nu^2\sigma_u^2\Bigg\} \end{aligned} \tag{OA.80}$$

Simplify the coefficient on $\mathbb{F}_t[\mu]$:

$$B^{(h-1)}(1-\nu) + \nu B^{(h-1)} + \phi^{h-1} = B^{(h-1)} + \phi^{h-1} \tag{OA.81}$$

For the variance terms, expand and simplify:

$$\begin{aligned} &-\frac{1}{2}\gamma^2\sigma_u^2 + \frac{1}{2}(\nu B^{(h-1)} + \phi^{h-1})^2\sigma_u^2 - \gamma(\nu B^{(h-1)} + \phi^{h-1})\sigma_u^2 + \frac{1}{2}\gamma^2\sigma_u^2 + \frac{1}{2}(\nu B^{(h-1)} + \phi^{h-1})^2\nu^2\sigma_u^2 \\ &= \frac{1}{2}(\nu B^{(h-1)} + \phi^{h-1})[(\nu B^{(h-1)} + \phi^{h-1})(1+\nu^2) - 2\gamma]\sigma_u^2 \end{aligned} \tag{OA.82}$$

Next, approximate by using $(1+\nu^2) \approx 1$ for small ν, yielding:

$$\frac{1}{2}(\nu B^{(h-1)} + \phi^{h-1})[(\nu B^{(h-1)} + \phi^{h-1}) - 2\gamma]\sigma_u^2 \tag{OA.83}$$

Define $C^{(h)} \equiv \nu B^{(h-1)} + \phi^{h-1}$. Then:

$$P_t^{(h)} = \exp\left\{A^{(h)} + B^{(h)}\mathbb{F}_t[\mu] + \phi^h e_t\right\} \tag{OA.84}$$

where:

$$A^{(h)} = A^{(h-1)} - r_f + \frac{1}{2}C^{(h)}[C^{(h)} - 2\gamma]\sigma_u^2 \tag{OA.85}$$

$$B^{(h)} = B^{(h-1)} + \phi^{h-1} = \frac{1-\phi^h}{1-\phi} \tag{OA.86}$$

$$C^{(h)} = \nu B^{(h-1)} + \phi^{h-1} \tag{OA.87}$$

with initial conditions $A^{(0)} = B^{(0)} = C^{(0)} = 0$.

OB.3.6 Aggregate Returns

Realized Strip Returns The realized return on the h-period strip from t to $t+1$ is:

$$R_{t+1}^{(h)} = \frac{P_{t+1}^{(h-1)}}{P_t^{(h)}} = \frac{\exp\left\{A^{(h-1)} + B^{(h-1)}\mathbb{F}_{t+1}[\mu] + \phi^{h-1}e_{t+1}\right\}}{\exp\left\{A^{(h)} + B^{(h)}\mathbb{F}_t[\mu] + \phi^h e_t\right\}} \tag{OA.88}$$

Substitute $\mathbb{F}_{t+1}[\mu] = (1-\nu)\mathbb{F}_t[\mu] + \nu(\mu + u_{t+1})$ and $e_{t+1} = \mu + \phi e_t + u_{t+1}$:

$$R_{t+1}^{(h)} = \exp\left\{(A^{(h-1)} - A^{(h)}) + (B^{(h-1)}(1-\nu) - B^{(h)})\mathbb{F}_t[\mu] + (\nu B^{(h-1)} + \phi^{h-1})(\mu + u_{t+1})\right\} \tag{OA.89}$$

Note that $B^{(h)} = B^{(h-1)} + \phi^{h-1}$, so:

$$B^{(h-1)}(1-\nu) - B^{(h)} = B^{(h-1)}(1-\nu) - B^{(h-1)} - \phi^{h-1} = -\nu B^{(h-1)} - \phi^{h-1} = -C^{(h)} \tag{OA.90}$$

Therefore:

$$R_{t+1}^{(h)} = \exp\left\{(A^{(h-1)} - A^{(h)}) + C^{(h)}(\mu - \mathbb{F}_t[\mu] + u_{t+1})\right\} \tag{OA.91}$$

Expected Strip Returns Taking expectations at time t:

$$\begin{aligned}\mathbb{F}_t[R_{t+1}^{(h)}] &= \mathbb{F}_t\left[\exp\left\{A^{(h-1)} - A^{(h)} + C^{(h)}(\mu - \mathbb{F}_t[\mu]) + C^{(h)}u_{t+1}\right\}\right] \\ &= \exp\left\{A^{(h-1)} - A^{(h)} + 0 + \frac{1}{2}C^{(h)2}\sigma_u^2\right\}\end{aligned} \tag{OA.92}$$

Substitute $A^{(h)} = A^{(h-1)} - r_f + \frac{1}{2}C^{(h)}[C^{(h)} - 2\gamma]\sigma_u^2$ and simplify:

$$\begin{aligned}\mathbb{F}_t[R_{t+1}^{(h)}] &= \exp\left\{A^{(h-1)} - A^{(h)} + \frac{1}{2}C^{(h)2}\sigma_u^2\right\} \\ &= \exp\left\{r_f + C^{(h)}\gamma\sigma_u^2\right\}\end{aligned} \tag{OA.93}$$

Aggregate Stock Price and Returns The aggregate stock price is the sum of strip prices:

$$P_t = \sum_{h=1}^{\infty} P_t^{(h)} \tag{OA.94}$$

The aggregate stock return is the value-weighted average:

$$R_{t+1} = \frac{\sum_{h=1}^{\infty} P_{t+1}^{(h-1)}}{\sum_{h=1}^{\infty} P_t^{(h)}} = \sum_{h=1}^{\infty} w_{t,h}R_{t+1}^{(h)}, \quad w_{t,h} = \frac{P_t^{(h)}}{\sum_{k=1}^{\infty} P_t^{(k)}} \tag{OA.95}$$

Constant Weights Approximation I assume that, under subjective beliefs, expected strip weights are approximately constant: $w_{t+j-1,h} \approx w_{t,h}$. Justification: In the small-gain limit as $\nu \to 0$, we have $C^{(h)} \approx \phi^{h-1}$. The strip price becomes:

$$P_t^{(h)} \approx \exp\left\{\tilde{A}^{(h)} + \tilde{B}^{(h)}\mathbb{F}_t[\mu] + \phi^h e_t\right\} \tag{OA.96}$$

where the coefficients depend on ϕ but not on ν (to first order). When e_t changes to e_{t+1}, all strip prices change by approximately $\exp\{\phi^h \Delta e_{t+1}\}$. Since this factor differs only by the power ϕ^h across maturities, and $\phi < 1$, the relative weights:

$$w_{t,h} = \frac{P_t^{(h)}}{\sum_k P_t^{(k)}} \tag{OA.97}$$

remain approximately constant over time. This approximation becomes exact as $\nu \to 0$ and is accurate for small learning gains. Under this approximation, the expected return is:

$$\begin{aligned}\mathbb{F}_t[R_{t+j}] &\approx \sum_{h=1}^{\infty} w_{t,h}\mathbb{F}_t[\mathbb{F}_{t+1}[\ldots\mathbb{F}_{t+j-1}[R_{t+j}^{(h)}]]] \\ &= \sum_{h=1}^{\infty} w_{t,h}\exp\left\{r_f + C^{(h)}\gamma\sigma_u^2\right\}\end{aligned} \tag{OA.98}$$

OB.3.7 Firm-Level Strip Prices and Returns

Each firm's total value is the sum of expected discounted future cash flows:

$$P_{i,t} = \sum_{h=1}^{\infty} P_{i,t}^{(h)}, \quad P_{i,t}^{(h)} = \mathbb{F}_t[M_{t+1} P_{i,t+1}^{(h-1)}] = \mathbb{F}_t[M_{t+1} \dots \mathbb{F}_{t+h-1}[M_{t+h} E_{t+h} \widetilde{E}_{i,t+h}]] \tag{OA.99}$$

Assuming independence between aggregate and idiosyncratic components:

$$P_{i,t}^{(h)} = P_t^{(h)} \cdot \mathbb{F}_t[\dots \mathbb{F}_{t+h-1}[\widetilde{E}_{i,t+h}]] \tag{OA.100}$$

Idiosyncratic Expectations To compute $\mathbb{F}_t[\dots \mathbb{F}_{t+h-1}[\widetilde{E}_{i,t+h}]]$, start with the one-period-ahead expectation:

$$\begin{aligned}
\mathbb{F}_{t+h-1}[\widetilde{E}_{i,t+h}] &= \mathbb{F}_{t+h-1}[\exp\{\widetilde{e}_{i,t+h}\}] \\
&= \mathbb{F}_{t+h-1}[\exp\{\widetilde{\phi}\widetilde{e}_{i,t+h-1} + \widetilde{\mu}_i + v_{i,t+h}\}] \\
&= \exp\left\{\widetilde{\phi}\widetilde{e}_{i,t+h-1} + \mathbb{F}_{t+h-1}[\widetilde{\mu}_i] + \frac{1}{2}\sigma_v^2\right\}
\end{aligned} \tag{OA.101}$$

Then iterate backward. Note that $\mathbb{F}_{t+h-1}[\widetilde{\mu}_i]$ depends on future shocks through the learning rule:

$$\mathbb{F}_{t+h-1}[\widetilde{\mu}_i] = (1-\nu)\mathbb{F}_{t+h-2}[\widetilde{\mu}_i] + \nu(\widetilde{\mu}_i + v_{i,t+h-1}) \tag{OA.102}$$

Taking expectations at time $t+h-2$:

$$\mathbb{F}_{t+h-2}[\mathbb{F}_{t+h-1}[\widetilde{\mu}_i]] = \mathbb{F}_{t+h-2}[\widetilde{\mu}_i] \tag{OA.103}$$

Also, substitute $\widetilde{e}_{i,t+h-1} = \widetilde{\phi}\widetilde{e}_{i,t+h-2} + \widetilde{\mu}_i + v_{i,t+h-1}$. Since $\widetilde{\mu}_i$ is unknown to the agent, when taking $\mathbb{F}_{t+h-2}[\cdot]$, the term $\widetilde{\mu}_i$ is replaced by $\mathbb{F}_{t+h-2}[\widetilde{\mu}_i]$:

$$\begin{aligned}
&\mathbb{F}_{t+h-2}[\mathbb{F}_{t+h-1}[\widetilde{E}_{i,t+h}]] \\
&= \mathbb{F}_{t+h-2}\left[\exp\left\{\widetilde{\phi}(\widetilde{\phi}\widetilde{e}_{i,t+h-2} + \widetilde{\mu}_i + v_{i,t+h-1}) + \mathbb{F}_{t+h-1}[\widetilde{\mu}_i] + \frac{1}{2}\sigma_v^2\right\}\right] \\
&= \exp\left\{\widetilde{\phi}^2\widetilde{e}_{i,t+h-2} + (1+\widetilde{\phi})\mathbb{F}_{t+h-2}[\widetilde{\mu}_i] + \frac{1}{2}(1+\widetilde{\phi}^2)\sigma_v^2\right\}
\end{aligned} \tag{OA.104}$$

Continuing this backward recursion to time t:

$$\begin{aligned}
\mathbb{F}_t[\dots \mathbb{F}_{t+h-1}[\widetilde{E}_{i,t+h}]] &= \exp\left\{\widetilde{\phi}^h\widetilde{e}_{i,t} + \sum_{j=0}^{h-1}\widetilde{\phi}^j\mathbb{F}_t[\widetilde{\mu}_i] + \sum_{j=0}^{h-1}\frac{1}{2}\widetilde{\phi}^{2j}\sigma_v^2\right\} \\
&= \exp\left\{\widetilde{\phi}^h\widetilde{e}_{i,t} + \frac{1-\widetilde{\phi}^h}{1-\widetilde{\phi}}\mathbb{F}_t[\widetilde{\mu}_i] + \frac{1}{2}\frac{1-\widetilde{\phi}^{2h}}{1-\widetilde{\phi}^2}\sigma_v^2\right\}
\end{aligned} \tag{OA.105}$$

Firm Strip Returns The realized firm-level strip return is:

$$R_{i,t+1}^{(h)} = \frac{P_{i,t+1}^{(h-1)}}{P_{i,t}^{(h)}} = \frac{P_{t+1}^{(h-1)}\mathbb{F}_{t+1}[\dots \mathbb{F}_{t+h-1}[\widetilde{E}_{i,t+h}]]}{P_t^{(h)}\mathbb{F}_t[\dots \mathbb{F}_{t+h-1}[\widetilde{E}_{i,t+h}]]} = R_{t+1}^{(h)} \cdot \frac{\mathbb{F}_{t+1}[\dots \mathbb{F}_{t+h-1}[\widetilde{E}_{i,t+h}]]}{\mathbb{F}_t[\dots \mathbb{F}_{t+h-1}[\widetilde{E}_{i,t+h}]]} \tag{OA.106}$$

From the calculations above:

$$\frac{\mathbb{F}_{t+1}[\dots \mathbb{F}_{t+h-1}[\widetilde{E}_{i,t+h}]]}{\mathbb{F}_t[\dots \mathbb{F}_{t+h-1}[\widetilde{E}_{i,t+h}]]} = \exp\left\{\widetilde{\phi}^{h-1}\widetilde{e}_{i,t+1} + \sum_{j=0}^{h-2}\widetilde{\phi}^j\mathbb{F}_{t+1}[\widetilde{\mu}_i] + \sum_{j=0}^{h-2}\frac{1}{2}\widetilde{\phi}^{2j}\sigma_v^2 - \widetilde{\phi}^h\widetilde{e}_{i,t} - \sum_{j=0}^{h-1}\widetilde{\phi}^j\mathbb{F}_t[\widetilde{\mu}_i] - \sum_{j=0}^{h-1}\frac{1}{2}\widetilde{\phi}^{2j}\sigma_v^2\right\} \tag{OA.107}$$

Substitute $\widetilde{e}_{i,t+1} = \widetilde{\mu}_i + \widetilde{\phi}\widetilde{e}_{i,t} + v_{i,t+1}$ and $\mathbb{F}_{t+1}[\widetilde{\mu}_i] = (1-\nu)\mathbb{F}_t[\widetilde{\mu}_i] + \nu(\widetilde{\mu}_i + v_{i,t+1})$:

$$= \exp\left\{\widetilde{\phi}^{h-1}(\widetilde{\mu}_i + \widetilde{\phi}\widetilde{e}_{i,t} + v_{i,t+1}) + \sum_{j=0}^{h-2}\widetilde{\phi}^j((1-\nu)\mathbb{F}_t[\widetilde{\mu}_i] + \nu(\widetilde{\mu}_i + v_{i,t+1})) - \widetilde{\phi}^h\widetilde{e}_{i,t} - \sum_{j=0}^{h-1}\widetilde{\phi}^j\mathbb{F}_t[\widetilde{\mu}_i] - \frac{1}{2}\widetilde{\phi}^{2(h-1)}\sigma_v^2\right\}$$

$$= \exp\left\{\widetilde{\phi}^{h-1}v_{i,t+1} + \sum_{j=0}^{h-2}\widetilde{\phi}^j\nu(\widetilde{\mu}_i - \mathbb{F}_t[\widetilde{\mu}_i] + v_{i,t+1}) + \widetilde{\phi}^{h-1}(\widetilde{\mu}_i - \mathbb{F}_t[\widetilde{\mu}_i]) - \frac{1}{2}\widetilde{\phi}^{2(h-1)}\sigma_v^2\right\}$$

$$= \exp\left\{\widetilde{C}^{(h)}(\widetilde{\mu}_i - \mathbb{F}_t[\widetilde{\mu}_i] + v_{i,t+1}) - \frac{1}{2}\widetilde{\phi}^{2(h-1)}\sigma_v^2\right\} \tag{OA.108}$$

where:

$$\widetilde{C}^{(h)} = \widetilde{\phi}^{h-1} + \nu\sum_{j=0}^{h-2}\widetilde{\phi}^j = \widetilde{\phi}^{h-1} + \nu\frac{1-\widetilde{\phi}^{h-1}}{1-\widetilde{\phi}} \tag{OA.109}$$

Therefore:

$$R_{i,t+1}^{(h)} = R_{t+1}^{(h)}\exp\left\{\widetilde{C}^{(h)}(\widetilde{\mu}_i - \mathbb{F}_t[\widetilde{\mu}_i] + v_{i,t+1}) - \frac{1}{2}\widetilde{\phi}^{2(h-1)}\sigma_v^2\right\} \tag{OA.110}$$

Expected Firm Returns Taking expectations:

$$\begin{aligned}\mathbb{F}_t[R_{i,t+1}^{(h)}] &= \mathbb{F}_t[R_{t+1}^{(h)}]\mathbb{F}_t\left[\exp\left\{\widetilde{C}^{(h)}(\widetilde{\mu}_i - \mathbb{F}_t[\widetilde{\mu}_i] + v_{i,t+1}) - \frac{1}{2}\widetilde{\phi}^{2(h-1)}\sigma_v^2\right\}\right] \\ &= \mathbb{F}_t[R_{t+1}^{(h)}]\exp\left\{\widetilde{C}^{(h)}\mathbb{F}_t[\widetilde{\mu}_i - \mathbb{F}_t[\widetilde{\mu}_i]] + \frac{1}{2}(\widetilde{C}^{(h)})^2\sigma_v^2 - \frac{1}{2}\widetilde{\phi}^{2(h-1)}\sigma_v^2\right\} \\ &= \mathbb{F}_t[R_{t+1}^{(h)}]\exp\left\{\frac{1}{2}((\widetilde{C}^{(h)})^2 - \widetilde{\phi}^{2(h-1)})\sigma_v^2\right\} \\ &= \exp\left\{r_f + C^{(h)}\gamma\sigma_u^2 + \frac{1}{2}((\widetilde{C}^{(h)})^2 - \widetilde{\phi}^{2(h-1)})\sigma_v^2\right\}\end{aligned} \tag{OA.111}$$

The firm-level stock return is:

$$R_{i,t+1} = \sum_{h=1}^{\infty} w_{i,t,h}R_{i,t+1}^{(h)}, \quad w_{i,t,h} = \frac{P_{i,t}^{(h)}}{\sum_{k=1}^{\infty}P_{i,t}^{(k)}} \tag{OA.112}$$

I assume that, under subjective beliefs, expected strip weights are approximately constant as in the aggregate case $w_{i,t+j-1,h} \approx w_{i,t,h}$, which yields the expected firm-level stock return:

$$\mathbb{F}_t[R_{i,t+j}] \approx \sum_{h=1}^{\infty} w_{i,t,h}\exp\left\{r_f + C^{(h)}\gamma\sigma_u^2 + \frac{1}{2}((\widetilde{C}^{(h)})^2 - \widetilde{\phi}^{2(h-1)})\sigma_v^2\right\} \tag{OA.113}$$

OB.3.8 Firm Valuation

The firm's equilibrium stock price is:

$$\begin{aligned}P_{i,t}^{(h)} &= P_t^{(h)} \cdot \mathbb{F}_t[\ldots\mathbb{F}_{t+h-1}[\widetilde{E}_{i,t+h}]] \\ &= \exp\left\{A^{(h)} + B^{(h)}\mathbb{F}_t[\mu] + \phi^h e_t\right\} \times \exp\left\{\widetilde{\phi}^h\widetilde{e}_{i,t} + \frac{1-\widetilde{\phi}^h}{1-\widetilde{\phi}}\mathbb{F}_t[\widetilde{\mu}_i] + \frac{1}{2}\frac{1-\widetilde{\phi}^{2h}}{1-\widetilde{\phi}^2}\sigma_v^2\right\} \\ &= \exp\left\{A_i^{(h)} + B^{(h)}\mathbb{F}_t[\mu] + \widetilde{B}^{(h)}\mathbb{F}_t[\widetilde{\mu}_i] + \phi^h e_t + \widetilde{\phi}^h\widetilde{e}_{i,t}\right\}\end{aligned} \tag{OA.114}$$

where:

$$A_i^{(h)} = A^{(h)} + \frac{1}{2}\frac{1-\widetilde{\phi}^{2h}}{1-\widetilde{\phi}^2}\sigma_v^2, \quad \widetilde{B}^{(h)} = \frac{1-\widetilde{\phi}^h}{1-\widetilde{\phi}} \tag{OA.115}$$

Therefore, the total firm value is:

$$P_{i,t} = \sum_{h=1}^{\infty}P_{i,t}^{(h)} = \sum_{h=1}^{\infty}\exp\left\{A_i^{(h)} + B^{(h)}\mathbb{F}_t[\mu] + \widetilde{B}^{(h)}\mathbb{F}_t[\widetilde{\mu}_i] + \phi^h e_t + \widetilde{\phi}^h\widetilde{e}_{i,t}\right\} \tag{OA.116}$$

OB.3.9 Hiring Condition and Labor Market Equilibrium

Firms post vacancies until the marginal cost of hiring equals its marginal value:

$$\frac{\kappa}{q_t} = \frac{P_{i,t}}{L_{i,t+1}} \tag{OA.117}$$

where κ is the cost per vacancy posting, q_t is the vacancy filling rate, and $L_{i,t+1}$ denotes employment. Given values for $\kappa, \delta, B, \eta, P_{i,t}$ and initial values for employment $L_{i,0}$, one can construct the sequence of vacancies $V_{i,t}$, employment $L_{i,t+1}$, labor market tightness θ_t, vacancy filling rates q_t, and unemployment rate U_t by solving for the employment accumulation (5), firm valuation (51), and optimal hiring (52) equations under a Cobb-Douglas matching function (4).

1. Initialize labor market tightness: $\theta_t^{(0)} = 1$

2. At iteration s, use labor market tightness $\theta_t^{(s)}$ to construct vacancy filling rate by using the Cobb-Douglas matching function in equation (4):

$$q_t^{(s)} = B(\theta_t^{(s)})^{-\eta} \tag{OA.118}$$

3. Update each firm's employment policy using the hiring equation (52):

$$L_{i,t+1}^{(s)} = \frac{P_{i,t} q_t^{(s)}}{\kappa} \tag{OA.119}$$

   where $P_{i,t}$ is determined by the firm valuation equation (51) under the constant-gain learning rules in equations (34) and (35).

4. Update each firm's vacancy posting using the employment accumulation equation (5):

$$V_{i,t}^{(s)} = \frac{1}{q_t^{(s)}}(L_{i,t+1}^{(s)} - (1-\delta)L_{i,t}) \tag{OA.120}$$

5. Aggregate firm-level variables over the set of firms I:

$$V_t^{(s)} = \sum_{i \in I} V_{i,t}^{(s)}, \quad L_{t+1}^{(s)} = \sum_{i \in I} L_{i,t+1}^{(s)}, \quad U_t^{(s)} = 1 - \sum_{i \in I} L_{i,t} \tag{OA.121}$$

6. Update labor market tightness: $\theta_t^{(s+1)} = \frac{V_t^{(s)}}{U_t^{(s)}}$. Check convergence: $|\theta_t^{(s+1)} - \theta_t^{(s)}| < \varepsilon$ for some small tolerance $\varepsilon > 0$. If not, return to step 2 with the updated values.

OB.3.10 Model-Implied Decompositions

The time-series decomposition of the aggregate vacancy filling rate is:

$$\log q_t = \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[r_{t+j}] - \left[el_t + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta e_{t+j}] \right] - \rho^h \mathbb{F}_t\left[pe_{t+h}\right] \tag{OA.122}$$

where $el_{i,t} \equiv \log E_{i,t} - \log L_{i,t+1}$ is log earnings per worker. The cross-sectional decomposition of hiring rates uses deviations from means $\widetilde{x}_{i,t} = x_{i,t} - \frac{1}{I}\sum_i x_{i,t}$:

$$\widetilde{hl}_{i,t} = -\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\widetilde{r}_{i,t+j}] + \left[\widetilde{el}_{i,t} + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta \widetilde{e}_{i,t+j}] \right] + \rho^h \mathbb{F}_t\left[\widetilde{pe}_{i,t+h}\right] \tag{OA.123}$$

Under constant-gain learning ($\nu > 0$), beliefs deviate from truth: $\mu - \mathbb{F}_t[\mu] \neq 0$ and $\widetilde{\mu}_i - \mathbb{F}_t[\widetilde{\mu}_i] \neq 0$. These distortions drive fluctuations in expected cash flow growth through equations (OA.70) and (OA.72), amplifying the cash flow channel while the discount rate channel remains muted.

OB.4 Model of Constant-Gain Learning from Prices and Cash Flows

In this section, I introduce a model of hiring in which firms form subjective beliefs about cash flows and prices using a constant-gain learning rule. The evolving expectations shape firms' vacancy posting decisions and drive variation in hiring and vacancy filling rates. The model embeds belief distortions in a search-and-matching framework and generates decompositions that can match those estimated from the data.

Cash Flow Process Assume that the firm's cash flow process consists of aggregate and idiosyncratic components. Firm i's earnings at time t are given by:

$$E_{i,t} = \exp(e_{i,t}) = E_t \cdot \widetilde{E}_{i,t} \tag{OA.124}$$

where E_t represents the aggregate component and $\widetilde{E}_{i,t}$ captures firm-specific variation. The log aggregate earnings follow a random walk with drift:

$$\Delta e_t = \log a + \log \varepsilon_t, \quad \log \varepsilon_t \sim \mathcal{N}(-\tfrac{s^2}{2}, s^2) \tag{OA.125}$$

while the log idiosyncratic component evolves as:

$$\Delta \widetilde{e}_{i,t} = \log \widetilde{a}_i + \log \widetilde{\varepsilon}_{i,t}, \quad \log \widetilde{\varepsilon}_{i,t} \sim \mathcal{N}(-\tfrac{\widetilde{s}_i^2}{2}, \widetilde{s}_i^2) \tag{OA.126}$$

For simplicity, I assume that the aggregate ε_t and idiosyncratic $\widetilde{\varepsilon}_{i,t}$ are independently distributed, and that subjective beliefs preserve this independence.

Full Information Rational Expectations Under full information rational expectations, agents know the true drift and volatility parameters and form expectations using the true data generating process. Let $g_{i,t}^{RE}$ and $m_{i,t}^{RE}$ denote expected earnings and price growth for firm i under rational beliefs, decomposed into aggregate (g_t^{RE}, m_t^{RE}) and idiosyncratic ($\widetilde{g}_{i,t}^{RE}$, $\widetilde{m}_{i,t}^{RE}$) components. Under rational expectations, all growth expectations equal the corresponding true drift parameters: $g_t^{RE} = m_t^{RE} = a$ and $\widetilde{g}_{i,t}^{RE} = \widetilde{m}_{i,t}^{RE} = \widetilde{a}_i$. Assuming a risk-neutral discount factor β, this makes the price-earnings ratio $P_{i,t}/E_{i,t} = \beta g_{i,t}^{RE}/(1-\beta m_{i,t}^{RE})$ constant. By independence of shocks, firm-level expectations are:

$$g_{i,t}^{RE} = g_t^{RE} \cdot \widetilde{g}_{i,t}^{RE} = a \cdot \widetilde{a}_i, \tag{OA.127}$$
$$m_{i,t}^{RE} = m_t^{RE} \cdot \widetilde{m}_{i,t}^{RE} = a \cdot \widetilde{a}_i \tag{OA.128}$$

Subjective Expectations Under Constant-Gain Learning Suppose that agents do not observe the true drift terms a and $\widetilde{a}_i$ in the cash flow process, and the firms do not know how their stock price $P_{i,t}$ is determined. Instead, they form beliefs and update these beliefs recursively as new information arrives. Firms form subjective expectations about the aggregate and idiosyncratic components of both cash flow growth and stock price growth:

$$\mathbb{F}_t[E_{t+1}] = g_t E_t, \quad \mathbb{F}_t[P_{t+1}] = m_t P_t, \tag{OA.129}$$
$$\mathbb{F}_t[\widetilde{E}_{i,t+1}] = \widetilde{g}_{i,t} \widetilde{E}_{i,t}, \quad \mathbb{F}_t[\widetilde{P}_{i,t+1}] = \widetilde{m}_{i,t} \widetilde{P}_{i,t} \tag{OA.130}$$

where g_t, m_t denote expectations about growth in the aggregate component and $\widetilde{g}_{i,t}$ and $\widetilde{m}_{i,t}$ denote expectations about growth in the idiosyncratic component. Under the independence of aggregate and idiosyncratic shocks, beliefs about total firm-level growth can be written as:

$$\mathbb{F}_t[E_{i,t+1}] = g_{i,t} E_{i,t} = g_t \widetilde{g}_{i,t} \cdot E_t \widetilde{E}_{i,t} \tag{OA.131}$$
$$\mathbb{F}_t[P_{i,t+1}] = m_{i,t} P_{i,t} = m_t \widetilde{m}_{i,t} \cdot P_t \widetilde{P}_{i,t} \tag{OA.132}$$

where $g_{i,t} = g_t \widetilde{g}_{i,t}$ and $m_{i,t} = m_t \widetilde{m}_{i,t}$. I assume that firms employ constant-gain learning to update their expectations using the rule:

$$g_t = g_{t-1} + \nu \left(\frac{E_{t-1}}{E_{t-2}} - g_{t-1} \right), \quad m_t = m_{t-1} + \nu \left(\frac{P_{t-1}}{P_{t-2}} - m_{t-1} \right) \tag{OA.133}$$

for the aggregate components, and

$$\widetilde{g}_{i,t} = \widetilde{g}_{i,t-1} + \nu \left(\frac{\widetilde{E}_{i,t-1}}{\widetilde{E}_{i,t-2}} - \widetilde{g}_{i,t-1} \right), \quad \widetilde{m}_{i,t} = \widetilde{m}_{i,t-1} + \nu \left(\frac{\widetilde{P}_{i,t-1}}{\widetilde{P}_{i,t-2}} - \widetilde{m}_{i,t-1} \right) \tag{OA.134}$$

for the idiosyncratic components, where ν is the constant gain parameter that governs the speed of learning. Suppose the initial beliefs are set equal to the growth rates under full information rational expectations:

$$g_0 = m_0 = a, \quad \widetilde{g}_{i,0} = \widetilde{m}_{i,0} = \widetilde{a}_i \tag{OA.135}$$

Note that the current price and current cash flows do not enter the learning rule for $g_{i,t}$ and $m_{i,t}$. The belief updates incorporate information with a lag by using information only up to period $t-1$, which eliminates the simultaneity between prices and price growth expectations. The lag in the updating equation can be motivated by an information structure in which agents observe part of the lagged transitory shocks to stock price growth (Adam et al., 2016).

Under constant-gain learning, agents update their beliefs using a fixed gain, which causes past observations to receive exponentially decreasing weights. As a result, memory fades over time and beliefs never fully converge to rational expectations, even in a stationary environment (Nagel and Xu, 2021). This learning scheme has the advantage of allowing beliefs to remain responsive to structural changes in the data-generating process. Compared to ordinary least squares (OLS) learning, where the gain vanishes over time, constant-gain learning avoids the counterfactual implication of declining volatility in predicted variables, and is often more realistic in environments with potential regime shifts.

Constant-gain learning can be micro-founded in two complementary ways. First, when agents are internally rational but lack external knowledge of market dynamics, they optimally forecast next-period prices using past data (Adam et al., 2016). Alternatively, when agents learn from recent experience, as older generations pass and newer ones rely more on recent data, the aggregation of their belief updates approximates a constant-gain rule (Nagel and Xu, 2021).

For parsimony and interpretability, the updating rules use the same constant gain parameter ν across all components. This reflects a shared rate at which firms update beliefs about different components of prices and cash flows. Existing estimates of the constant gain parameter ν are deliberately small, meaning that learning is slow and allows subjective beliefs to remain persistently distorted even after observing large forecast errors (Malmendier and Nagel, 2015; Adam et al., 2016).[1] This persistence plays an important role for generating the sustained belief distortions needed to explain fluctuations in hiring and unemployment.

Subjective Firm Valuation To highlight how learning can improve the model's performance, I consider the simplest asset pricing model by assuming risk-neutral agents and time separable preferences (Adam et al., 2016). In this case, the aggregate stock price under subjective beliefs satisfies:

$$P_t = \beta \mathbb{F}_t[P_{t+1} + E_{t+1}] = \beta(m_t P_t + g_t E_t) \tag{OA.136}$$

which implies $P_t(1 - \beta m_t) = \beta g_t E_t$ and thus we have

$$P_t = \frac{\beta g_t}{1 - \beta m_t} \cdot E_t \tag{OA.137}$$

The firm's equilibrium stock price under subjective beliefs is:

$$P_{i,t} = \beta \mathbb{F}_t[P_{i,t+1} + E_{i,t+1}] = \beta(m_{i,t} P_{i,t} + g_{i,t} E_{i,t}) \tag{OA.138}$$

which implies $P_{i,t}(1 - \beta m_{i,t}) = \beta g_{i,t} E_{i,t}$ and thus we have

$$P_{i,t} = \frac{\beta g_{i,t}}{1 - \beta m_{i,t}} \cdot E_{i,t} \tag{OA.139}$$

where β is the time discount factor. The equation shows that the firm's value rises with expected cash flow growth $g_{i,t}$ and falls with expected price growth $m_{i,t}$. The belief distortions captured in these expectation terms will affect the firm's hiring decisions through its valuation.

[1] The constant-gain learning specification for cash flow growth is supported by empirical evidence showing that survey respondents update their long-run earnings expectations only gradually following short-term earnings surprises (Nagel and Xu, 2021; De La O et al., 2024). The learning specification for stock price growth is motivated by empirical evidence showing that the implied return expectation can reproduce the dynamics of various survey based measures of subjective return expectations (Adam et al., 2016).

Projection Facility To prevent agents from having an infinite demand for stocks based on the valuations in (OA.137) and (OA.139), I assume that the subjective beliefs about price growth are bounded such that

$$0 < m_t < \beta^{-1}, \quad 0 < m_{i,t} = m_t \widetilde{m}_{i,t} < \beta^{-1} \tag{OA.140}$$

which rules out the case $m_{i,t} \geq \beta^{-1}$ where the expected stock returns are greater than the inverse of the time discount factor. To prevent perceived stock price growth from violating the bounds in (OA.140), I apply a projection facility which makes a smooth modification to the belief-updating equation (Timmermann, 1993; Cogley and Sargent, 2005; Adam et al., 2016). If the updated belief from (OA.133) exceeds a constant $m^U \leq \beta^{-1}$, then the update is ignored:

$$m_t = m_{t-1} \quad \text{if } m_{t-1} + \nu \left(\frac{P_{t-1}}{P_{t-2}} - m_{t-1} \right) \geq m^U. \tag{OA.141}$$

For the idiosyncratic component, the bound applies to the firm-level expectation $m_{i,t} = m_t \widetilde{m}_{i,t}$. Given beliefs about the aggregate component m_t, the projection rule therefore becomes:

$$\widetilde{m}_{i,t} = \widetilde{m}_{i,t-1} \quad \text{if } \; m_t \left[\widetilde{m}_{i,t-1} + \nu \left(\frac{\widetilde{P}_{i,t-1}}{\widetilde{P}_{i,t-2}} - \widetilde{m}_{i,t-1} \right) \right] \geq m^U \tag{OA.142}$$

This procedure can be interpreted as an approximate Bayesian updating scheme where agents have a truncated prior that assigns probability zero to $m_t \geq m^U$ and $m_{i,t} \geq m^U$ (Adam et al., 2016). It can be viewed as agents ignoring observations that would lead to beliefs implying infinite demand for stocks, which would represent economically implausible behavior.

Applying the projection facility is equivalent to imposing that firm-level price-earnings ratios remain below an upper bound $U^{PE} \equiv \beta a/(1 - \beta m^U)$. One interpretation is that, if the price-earnings ratio exceeds this upper bound, either market participants begin to fear a sharp downturn or some regulatory authority intervenes to bring prices down. In the simulations below, the results are not sensitive to the exact value of U^{PE} provided it is sufficiently high, since the bounding facility binds only rarely.

Hiring Condition I close the model by connecting asset valuations to firm hiring behavior. The connection to labor markets operates through the hiring condition. Firms post vacancies until the marginal cost of hiring equals its marginal value:

$$\underbrace{\frac{\kappa}{q_t}}_{\text{Cost of Hiring}} = \underbrace{\frac{P_{i,t}}{L_{i,t+1}}}_{\text{Value of Hiring}} \tag{OA.143}$$

where κ is the cost per vacancy, q_t is the vacancy filling rate, and $L_{i,t+1}$ represents future employment. When firms are overly pessimistic about their expected cash flows (low $g_{i,t}$), this leads to lower firm value $P_{i,t}$, which reduces the value of hiring and leads to fewer job postings. The resulting decrease in vacancy creation drives up unemployment and reduces the vacancy filling rate q_t.

Let $el_{i,t} \equiv e_{i,t} - l_{i,t+1} = \log E_{i,t} - \log L_{i,t+1}$ denote log earnings per worker. Given values for $\kappa, \delta, B, \eta, P_{i,t}$ and initial values for employment $L_{i,0}$, one can construct the sequence of vacancies $V_{i,t}$, employment $L_{i,t+1}$, labor market tightness θ_t, vacancy filling rates q_t, and unemployment rate U_t by solving for the firm valuation, optimal hiring, and employment accumulation equations:

1. Initialize labor market tightness: $\theta_t^{(0)} = 1$

2. At iteration s, construct vacancy filling rate under Cobb-Douglas matching:

$$q_t^{(s)} = B(\theta_t^{(s)})^{-\eta} \tag{OA.144}$$

3. Update each firm's employment policy using the hiring equation (OA.143):

$$L_{i,t+1}^{(s)} = \frac{P_{i,t} q_t^{(s)}}{\kappa} \tag{OA.145}$$

   where $P_{i,t}$ is determined by the firm valuation equation (OA.139) under the constant-gain learning rules in (OA.133) and (OA.134).

4. Update each firm's vacancy posting using the employment accumulation:

$$V_{i,t}^{(s)} = \frac{1}{q_t^{(s)}}(L_{i,t+1}^{(s)} - (1-\delta)L_{i,t}) \tag{OA.146}$$

5. Aggregate firm-level variables over the set of firms I:

$$P_t = \sum_{i\in I} P_{i,t}, \quad V_t^{(s)} = \sum_{i\in I} V_{i,t}^{(s)}, \quad L_{t+1}^{(s)} = \sum_{i\in I} L_{i,t+1}^{(s)}, \quad U_t^{(s)} = 1 - \sum_{i\in I} L_{i,t} \tag{OA.147}$$

6. Update labor market tightness: $\theta_t^{(s+1)} = \frac{V_t^{(s)}}{U_t^{(s)}}$. Check convergence: $|\theta_t^{(s+1)} - \theta_t^{(s)}| < \varepsilon$ for some small tolerance $\varepsilon > 0$. If not, return to step 2 with the updated values.

Long-Horizon Cash Flow Growth and Stock Returns In this learning environment, the realized $j \geq 1$ period ahead log cash flow growth $\Delta e_{i,t+j} \equiv \log(E_{i,t+j}/E_{i,t+j-1})$ follows:

$$\Delta e_{i,t+j} = \log a_i + \log \varepsilon_{i,t+j} \tag{OA.148}$$

and stock returns $r_{i,t+j} \equiv \log((P_{i,t+j} + E_{i,t+j})/P_{i,t+j-1})$ follow:

$$r_{i,t+j} = \log\left(\frac{E_{i,t+j}}{E_{i,t+j-1}} \frac{E_{i,t+j-1}}{P_{i,t+j-1}} \left(\frac{P_{i,t+j}}{E_{i,t+j}} + 1\right)\right) \tag{OA.149}$$

$$= \Delta e_{i,t+j} + \log\left(\frac{1-\beta m_{i,t+j-1}}{\beta g_{i,t+j-1}}\right) + \log\left(\frac{1-\beta m_{i,t+j} + \beta g_{i,t+j}}{1-\beta m_{i,t+j}}\right) \tag{OA.150}$$

where $a_i \equiv a \cdot \widetilde{a}_i$ and $\varepsilon_{i,t} \equiv \varepsilon_t \cdot \widetilde{\varepsilon}_{i,t}$. The price-earnings ratios are based on the firm valuations implied by equation (OA.139).

Subjective expectations of these variables reflect beliefs about future earnings and capital gains. In models with constant-gain learning, beliefs evolve with fading memory, breaking the law of iterated expectations and making resale and buy-and-hold valuation methods non-equivalent. The buy-and-hold approach evaluates long-run payoffs under today's beliefs, while the resale method prices assets through a sequence of one-period-ahead valuations, each using updated beliefs. Following Nagel and Xu (2021), I adopt the resale valuation approach because it ensures time consistency under belief updating and reflects the idea that assets are effectively resold across agents with evolving expectations. I assume that the manager and the representative investor share the same beliefs and both apply the resale method, ensuring consistency between decision-making and valuation.

Let x_t and $\widetilde{x}_{i,t}$ denote the aggregate and idiosyncratic level of a variable $x \in \{E, P\}$ at time t, which are either aggregate cash flows or prices. Define:

$$R_t^x \equiv \frac{x_t}{x_{t-1}}, \quad Z_t^x \equiv (1-\nu)Z_{t-1}^x + \nu R_{t-1}^x \tag{OA.151}$$

$$\widetilde{R}_{i,t}^x \equiv \frac{\widetilde{x}_{i,t}}{\widetilde{x}_{i,t-1}}, \quad \widetilde{Z}_{i,t}^x \equiv (1-\nu)\widetilde{Z}_{i,t-1}^x + \nu \widetilde{R}_{i,t-1}^x \tag{OA.152}$$

That is, $Z_t^x, \widetilde{Z}_{i,t}^x$ denotes the subjective expectation of the growth rate of variable x, formed using constant-gain learning based on past realized growth $R_{t-1}^x, \widetilde{R}_{i,t-1}^x$, respectively. It can be shown by induction that the j-step-ahead expectation at time t is given by:

$$\mathbb{F}_t[Z_{t+j}^x] = a_j^x Z_t^x + b_j^x R_t^x, \tag{OA.153}$$

$$\mathbb{F}_t[\widetilde{Z}_{i,t+j}^x] = a_j^x \widetilde{Z}_{i,t}^x + b_j^x \widetilde{R}_{i,t}^x, \tag{OA.154}$$

$$\mathbb{F}_t[Z_{i,t+j}^x] = \mathbb{F}_t[Z_{t+j}^x] \cdot \mathbb{F}_t[\widetilde{Z}_{i,t+j}^x] \tag{OA.155}$$

with recursively defined coefficients:

$$a_0 = 1, \quad b_0 = 0, \quad a_1 = 1-\nu, \quad b_1 = \nu, \tag{OA.156}$$

$$a_j = (1-\nu)a_{j-1} + \nu a_{j-2}, \quad b_j = (1-\nu)b_{j-1} + \nu b_{j-2}, \quad j \geq 2 \tag{OA.157}$$

Base case ($j = 0$). At time t, the value Z_t^x is known:

$$\mathbb{F}_t[Z_t^x] = Z_t^x = a_0 Z_t^x + b_0 R_t^x. \tag{OA.158}$$

Base case ($j = 1$). From the learning rule:

$$Z_{t+1}^x = (1 - \nu)Z_t^x + \nu R_t^x, \tag{OA.159}$$

Taking expectations at time t:

$$\mathbb{F}_t[Z_{t+1}^x] = (1 - \nu)Z_t^x + \nu R_t^x = a_1 Z_t^x + b_1 R_t^x. \tag{OA.160}$$

Inductive step. Assume for $j - 1$ and $j - 2$ that:

$$\mathbb{F}_t[Z_{t+j-1}^x] = a_{j-1} Z_t^x + b_{j-1} R_t^x, \tag{OA.161}$$

$$\mathbb{F}_t[Z_{t+j-2}^x] = a_{j-2} Z_t^x + b_{j-2} R_t^x. \tag{OA.162}$$

Then, by the learning rule:

$$Z_{t+j}^x = (1 - \nu)Z_{t+j-1}^x + \nu R_{t+j-1}^x. \tag{OA.163}$$

Taking expectations at time t, note that Z_{t+j-2}^x is defined as the time-$(t + j - 2)$ forecast of R_{t+j-1}^x. Since the innovation in R_{t+j-1}^x realized at $t + j - 1$ is mean-independent of information available at t, the time-t expectation of R_{t+j-1}^x coincides with the time-t expectation of Z_{t+j-2}^x:

$$\mathbb{F}_t[R_{t+j-1}^x] = \mathbb{F}_t[Z_{t+j-2}^x] \tag{OA.164}$$

which implies:

$$\mathbb{F}_t[Z_{t+j}^x] = (1 - \nu)\mathbb{F}_t[Z_{t+j-1}^x] + \nu \mathbb{F}_t[Z_{t+j-2}^x] \tag{OA.165}$$

$$= (1 - \nu)(a_{j-1} Z_t^x + b_{j-1} R_t^x) + \nu(a_{j-2} Z_t^x + b_{j-2} R_t^x) \tag{OA.166}$$

$$= [(1 - \nu)a_{j-1} + \nu a_{j-2}]\, Z_t^x + [(1 - \nu)b_{j-1} + \nu b_{j-2}]\, R_t^x. \tag{OA.167}$$

Thus, the recursion holds for j, completing the induction. After making a first-order approximation $\mathbb{F}_t[\log(X)] \approx \log(\mathbb{F}_t[X])$, subjective expectations of log cash flow growth can be written as:

$$\mathbb{F}_t[\Delta e_{t+j}] = \mathbb{F}_t\left[\log\left(\frac{E_{t+j}}{E_{t+j-1}}\right)\right] \approx \log\left(\mathbb{F}_t\left[\frac{E_{t+j}}{E_{t+j-1}}\right]\right) = \log\left(\mathbb{F}_t\left[g_{t+j-1}\right]\right) \tag{OA.168}$$

$$\mathbb{F}_t[\Delta e_{i,t+j}] = \mathbb{F}_t\left[\log\left(\frac{E_{i,t+j}}{E_{i,t+j-1}}\right)\right] \approx \log\left(\mathbb{F}_t\left[\frac{E_{i,t+j}}{E_{i,t+j-1}}\right]\right) = \log\left(\mathbb{F}_t\left[g_{i,t+j-1}\right]\right) \tag{OA.169}$$

Similarly, subjective expectations of log stock returns can be written as:

$$\begin{aligned}
\mathbb{F}_t[r_{t+j}] &= \mathbb{F}_t\left[\Delta e_{t+j} + \log\left(\frac{1 - \beta m_{t+j-1}}{\beta g_{t+j-1}}\right) + \log\left(\frac{1 - \beta m_{t+j} + \beta g_{t+j}}{1 - \beta m_{t+j}}\right)\right] \\
&\approx \log\left(\mathbb{F}_t[g_{t+j-1}]\right) + \log\left(\frac{1 - \beta\mathbb{F}_t[m_{t+j-1}]}{\beta\mathbb{F}_t[g_{t+j-1}]}\right) + \log\left(\frac{1 - \beta\mathbb{F}_t[m_{t+j}] + \beta\mathbb{F}_t[g_{t+j}]}{1 - \beta\mathbb{F}_t[m_{t+j}]}\right) \\
&\approx (1 - \beta)\log\left(\mathbb{F}_t[g_{t+j-1}]\right) + \log\left(\frac{1 - \beta\mathbb{F}_t[m_{t+j-1}]}{1 - \beta\mathbb{F}_t[m_{t+j}]}\right) + \log\left(1 - \beta\mathbb{F}_t[m_{t+j}] + \beta\mathbb{F}_t[g_{t+j}]\right)
\end{aligned} \tag{OA.170}$$

$$\begin{aligned}
\mathbb{F}_t[r_{i,t+j}] &= \mathbb{F}_t\left[\Delta e_{i,t+j} + \log\left(\frac{1 - \beta m_{i,t+j-1}}{\beta g_{i,t+j-1}}\right) + \log\left(\frac{1 - \beta m_{i,t+j} + \beta g_{i,t+j}}{1 - \beta m_{i,t+j}}\right)\right] \\
&\approx \log\left(\mathbb{F}_t[g_{i,t+j-1}]\right) + \log\left(\frac{1 - \beta\mathbb{F}_t[m_{i,t+j-1}]}{\beta\mathbb{F}_t[g_{i,t+j-1}]}\right) + \log\left(\frac{1 - \beta\mathbb{F}_t[m_{i,t+j}] + \beta\mathbb{F}_t[g_{i,t+j}]}{1 - \beta\mathbb{F}_t[m_{i,t+j}]}\right) \\
&\approx (1 - \beta)\log\left(\mathbb{F}_t[g_{i,t+j-1}]\right) + \log\left(\frac{1 - \beta\mathbb{F}_t[m_{i,t+j-1}]}{1 - \beta\mathbb{F}_t[m_{i,t+j}]}\right) + \log\left(1 - \beta\mathbb{F}_t[m_{i,t+j}] + \beta\mathbb{F}_t[g_{i,t+j}]\right)
\end{aligned} \tag{OA.171}$$

where $\mathbb{F}_t[g_{i,t+j}]$ and $\mathbb{F}_t[m_{i,t+j}]$ are determined by the recursion in equations (OA.155) through (OA.157). Under constant-gain learning, realized stock returns $r_{i,t+j}$ and expected cash flow growth $\mathbb{F}_t[\Delta e_{i,t+j}]$ can fluctuate substantially due to large and persistent distortions in subjective beliefs embedded in $g_{i,t}$. In contrast, expected stock returns $\mathbb{F}_t[r_{i,t+j}]$ from equation (OA.171) can show only small fluctuations because its variation depends mainly on the gap between expected cash flow growth and price growth $g_{i,t} - m_{i,t}$. Since β is a number close to one, the first term in equation (OA.171) involving $1 - \beta$ will be quantitatively small. Since the learning rate ν is small, the one-period belief revisions in $\mathbb{F}_t[m_{i,t+j}]$ will also be quantitatively small in equation (OA.171). Since both $g_{i,t}$ and $m_{i,t}$ terms adjust slowly and often move together, their difference remains relatively stable. This generates the empirically observed pattern of high volatility in realized returns but low volatility in expected returns, consistent with survey evidence on return expectations.

Model-Implied Decompositions I use data simulated from the learning model to decompose the vacancy filling rate at the aggregate level and hiring rates at the firm level. The time-series decomposition of the aggregate vacancy filling rate q_t is given by:

$$\log q_t = \underbrace{\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[r_{t+j}]}_{\text{Discount Rate}} - \underbrace{\left[el_t + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta e_{t+j}] \right]}_{\text{Cash Flow}} - \underbrace{\rho^h \mathbb{F}_t \left[pe_{t+h} \right]}_{\text{Future Price-Earnings}} \tag{OA.172}$$

where $x_t = \sum_{i \in I} x_{i,t}$ aggregates firm-level variable $x_{i,t}$. To analyze differences across firms, I estimate a cross-sectional decomposition of hiring rates using simulated firm-level data:

$$\widetilde{hl}_{i,t} = \underbrace{-\sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\widetilde{r}_{i,t+j}]}_{\text{Discount Rate}} + \underbrace{\left[\widetilde{el}_{i,t} + \sum_{j=1}^{h} \rho^{j-1} \mathbb{F}_t[\Delta \widetilde{e}_{i,t+j}] \right]}_{\text{Cash Flow}} + \underbrace{\rho^h \mathbb{F}_t \left[\widetilde{pe}_{i,t+h} \right]}_{\text{Future Price-Earnings}} \tag{OA.173}$$

where $\widetilde{x}_{i,t} = x_{i,t} - \frac{1}{I} \sum_i x_{i,t}$ denotes a cross-sectional deviation from the mean at time t.

Firms' hiring decisions reflect their evolving beliefs about cash flow growth $g_{i,t}$ and stock price growth $m_{i,t}$, which are updated according to the constant-gain learning rules. The slow learning rate in the model can generate large and persistent fluctuations in $g_{i,t}$ which drives fluctuations in expected cash flow growth $\mathbb{F}_t[\Delta e_{i,t+j}]$ and realized stock returns $r_{i,t+j}$. In contrast, the model can produce a low volatility in expected returns $\mathbb{F}_t[r_{i,t+j}]$ because their variation depends only on the gap between cash flow growth and price growth $g_{i,t} - m_{i,t}$, which is relatively stable over time. Therefore under subjective beliefs, the cash flow component in the decompositions will be highly volatile while the discount rate component remains relatively muted. Consequently, subjective expectations will systematically over-weight the role of cash flows relative to discount rates, generating the empirical pattern observed in the data. This contrasts sharply with rational expectations where the cash flow component contributes zero to the variance because expected future cash flow growth equals the constant drift term.

Simulation Details I simulate a panel of 300 firms over 500 periods, where the first 150 periods are discarded as a burn-in to eliminate the influence of initial conditions. Under constant-gain learning, each firm updates its beliefs using the updating rules in equations (OA.133) and (OA.134). All expectations, returns, and decompositions are computed at a monthly frequency using the model equations derived above. At each horizon h, I compute the model-implied time-series decomposition of the aggregate vacancy filling rate based on equation (OA.172) and the cross-sectional decomposition of the firm-level hiring rates (OA.173). I then compare these model-implied decompositions to those estimated from the observed data.

Model Estimation Table OA.9 reports the parameters used in the quantitative model along with the empirical moments they are calibrated to or sourced from. The drift a and volatility s of aggregate cash flow growth is set to match the long-run mean and standard deviation of aggregate U.S. dividend growth (Adam et al., 2016). The drift $\widetilde{a}$ and volatility $\widetilde{s}_i$ of idiosyncratic earnings growth is set to match the long-run mean and standard deviation of dividend growth across listed firms. The time discount rate $\rho = \exp(\overline{pe})/(1+\exp(\overline{pe}))$ is chosen to be consistent with a steady-state price-earnings ratio from the Campbell and Shiller (1988) present value identity, where $\overline{pe}$ is the long-run average of the log price-earnings ratio over 1983–2023.

The speed at which agents discount past observations of realized cash flow growth depends on the constant gain parameter ν in the learning rule. This parameter shapes the persistence and volatility of the price-earnings ratio and the extent of return predictability. I take the value directly from survey-based estimates in Malmendier and Nagel (2015), setting it to $\nu = 0.018$ at the quarterly frequency. This implies that in forming expectations, agents assign a weight of 0.018 to the most recent growth surprise and $1 - \nu = 0.982$ to their previous estimate, making the perceived growth rate evolve slowly over time.

Labor market parameters are mainly adopted from Kehoe et al. (2023). Following Shimer (2005), I normalize the value of labor market tightness θ to one in the deterministic steady state, which implies an efficiency of the matching function $B = 0.562$ by noting from the matching function that $q = B\theta^{-\eta}$. I set the elasticity of the matching function to $\eta = 0.5$ following Ljungqvist and Sargent (2017). I use an annual job separation rate of $\delta = 0.286$, which is the annualized value of the Abowd-Zellner corrected estimate by Krusell et al. (2017) based on data from the Current Population Survey (CPS). Following Elsby and Michaels (2013), per-worker vacancy posting cost 0.314 is targeted to match a per-worker hiring cost κ/q equal to 14 percent of the quarterly worker compensation. In the context of the annual calibration of this model, this implies a value approximately equal $\kappa = 4 \times 0.14 \times q = 0.314$, where 4×0.14 is the annualized percent of worker compensation, while $q = 0.562$ is the long-run average of the vacancy filling rate in the historical sample from 1983 to 2023.

Table OA.9: Model Parameters

Parameter	Value	Moments
ν	0.006	Constant-gain learning rate (Malmendier and Nagel (2015))
a	1.009	Mean of U.S. aggregate earnings growth
s	0.025	S.D. of U.S. aggregate earnings growth
$\tilde{a}_i$	1.000	Mean of U.S. idiosyncratic earnings growth
$\tilde{s}_i$	0.036	S.D. of U.S. idiosyncratic earnings growth
β	0.996	Time discount rate (Adam et al. (2016))
ρ	0.998	Average price-earnings ratio
B	0.562	Matching function efficiency (Kehoe et al. (2023))
η	0.500	Matching function elasticity (Kehoe et al. (2023))
δ	0.028	Separation rate (Kehoe et al. (2023))
κ	0.026	Per worker hiring cost (Elsby and Michaels (2013))

Notes: Table reports the parameter values used in the quantitative model along with the empirical moments they are calibrated to or sourced from. The model is calibrated at a monthly frequency.

Model vs. Data: Variance Decompositions The model successfully replicates the empirical variance decompositions from the data. Figure OA.13 shows that the model can reproduce the finding that belief distortions drive excess sensitivity to cash flow news in explaining labor market fluctuations.

Panel (a) presents the time-series decomposition of the vacancy filling rate, comparing contributions under subjective and rational expectations. The model captures the empirical pattern where subjective expectations (dark bars) assign a larger role to cash flows compared to rational expectations (light bars). The model-implied values (circles and triangles) align closely with the empirical estimates, demonstrating the model's ability to match the data.

Panel (b) shows the cross-sectional decomposition of hiring rates across firms. Again, the model captures the empirical pattern that subjective belief distortions drive excess sensitivity to cash flow news. This cross-sectional fit is particularly important as it shows that the model can explain not just aggregate patterns but also the heterogeneity in hiring behavior across different firms.

Model vs. Data: Moments Table OA.10 demonstrates that the constant-gain learning model successfully matches both asset market and labor market moments. The table compares moments generated by the learning model against those generated from a rational model under no learning, where all agents have full information rational expectations. To generate simulations under the rational model, I employ the same sequence of shocks as in the baseline learning specification but set the learning rate parameter to zero. This eliminates belief updating and, conditional on the true initial values, reduces the model exactly to its rational expectations counterpart in equations (OA.127) and (OA.128).

Panel (a) reports time-series and cross-sectional moments for asset prices. The learning model broadly matches the mean and volatility of price-earnings ratios, the persistence in valuations, and the volatility of returns and expected returns. In contrast, the rational expectations model severely understates price-earnings volatility and generates virtually no variation in expected returns, confirming that belief distortions are essential for matching observed financial market behavior (Adam et al., 2016). For the cross-sectional moments, the learning model captures the dispersion in price-earnings ratios, expected earnings growth, returns, and expected returns. These moments confirm that the firm-specific beliefs $\widetilde{g}_{i,t}$ and $\widetilde{m}_{i,t}$ generate realistic heterogeneity in firm valuations and expectations. The rational expectations model, by construction, produces minimal cross-sectional variation in expectations, highlighting how constant-gain learning creates the belief heterogeneity observed in the data.

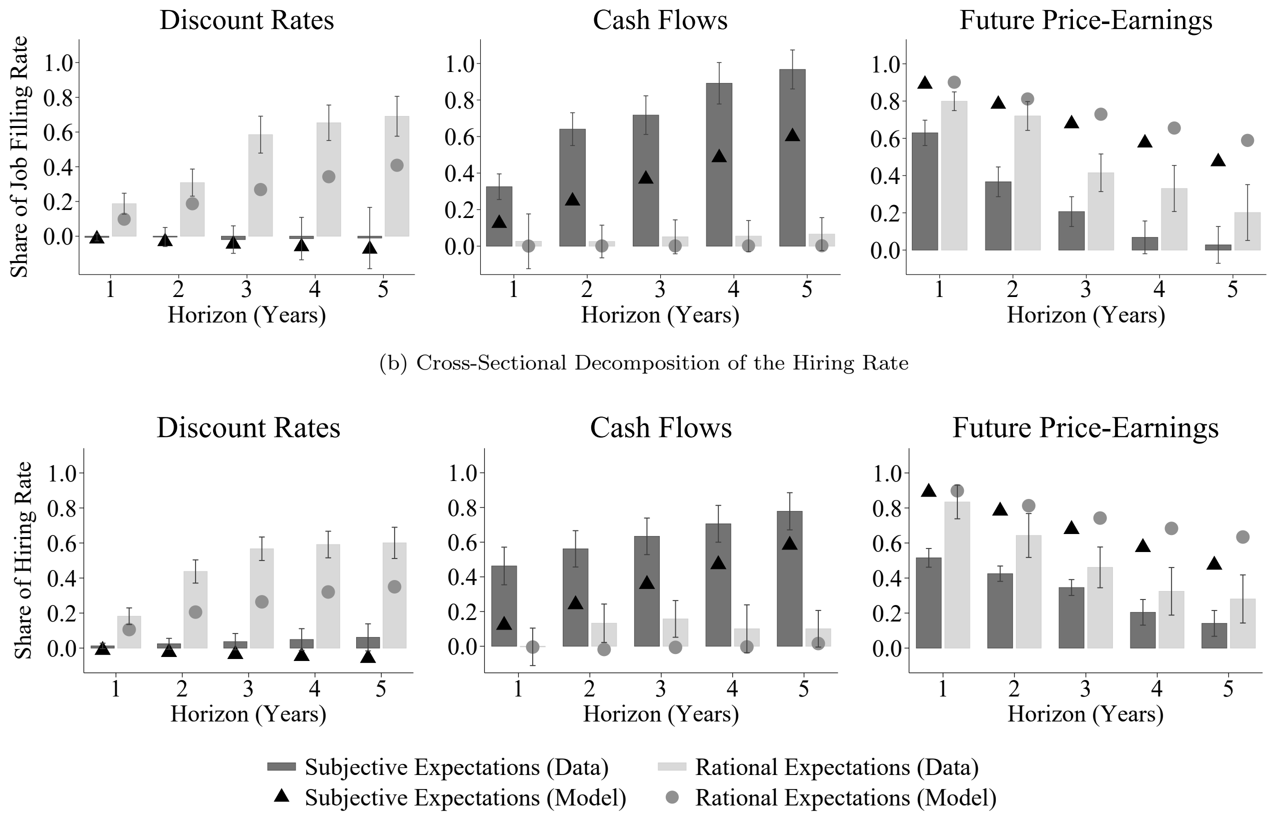
Panel (b) reports moments related to the labor market. The learning model broadly matches key labor market statistics including the volatility and persistence of the vacancy filling rate q_t and unemployment rate u_t. The constant-gain learning model only slightly undershoots the volatility of the unemployment rate, which is a substantial improvement over the rational expectations model where unemployment volatility is typically an order of magnitude too small. The learning model's ability to match these moments demonstrates that the constant-gain learning mechanism provides a coherent explanation for both asset market and labor market fluctuations.

Response to 1 Std. Dev. Shock to Cash Flow Growth Expectation To examine the dynamic implications of the model and compare them with the data, I estimate a four-variable VAR where the observation vector includes expected cash flow growth, expected returns, expected price-earnings, and the job-filling rate. The VAR is estimated using both the actual survey data and the simulated series generated from the model. For identification, I apply a recursive (Cholesky) scheme in which expected cash flow growth is ordered first, so that the estimated impulse responses trace out the effect of a one standard deviation shock to cash flow growth expectations. This identification strategy allows me to

Figure OA.13: Model vs. Data: Variance Decompositions

(a) Time-Series Decomposition of the Vacancy Filling Rate

(b) Cross-Sectional Decomposition of the Hiring Rate



Notes: Figure illustrates the discount rate, cash flow, and future price-earnings components of the time-series decomposition of the aggregate vacancy filling rate (panel (a)) and cross-sectional decomposition of the hiring rate (panel (b)). Light bars show contributions under rational expectations; dark bars show contributions under subjective expectations. The sample is quarterly from 2005Q1 to 2023Q4. Each bar shows Newey-West 95% confidence intervals with lags = 4. Circle and triangle dots show the values of rational and subjective expectations implied by the model, respectively.

interpret the innovation to expected cash flows as an exogenous shift in beliefs about future earnings growth. Figure OA.14 reports the resulting impulse response functions.

The impulse response functions in Figure OA.14 reveal several notable patterns. Expectations of cash flow growth jump immediately on impact and then gradually decay back toward zero. Subjective expected returns exhibit a positive, hump-shaped response that peaks with a lag before fading out. The mechanism behind this pattern is straightforward: a positive shock to expected cash flow growth raises expected stock returns and current stock prices. Because beliefs are updated with a lag, higher stock prices increase expected price growth in the following period, which in turn drives stock prices even higher. This feedback loop amplifies the initial shock for several periods, but each successive round of increases diminishes as the memory of the initial shock fades. Eventually, realized price growth begins to fall short of the inflated expectations, at which point the response of expected returns peaks and gradually declines. The subjective price-earnings ratio rises initially before decaying back to zero. Finally, the job-filling rate falls immediately after the shock and then slowly converges back to its baseline level.

Role of the Learning Rate ν The second figure examines the role of the constant-gain learning rate, ν, in shaping the variance decomposition of the job-filling rate. Figure OA.15 plots the share of job-filling rate variance explained by subjective discount rates on the left and by subjective cash flow expectations on the right, evaluated at a five-year horizon across a range of values for ν. For relatively small values of the learning rate, the decomposition is similar: discount-rate and cash-flow components contribute in roughly stable proportions. However, as the learning rate increases, the results diverge. A higher learning rate implies that agents place greater weight on recent observations, making expectations more responsive to new information but shortening the memory of past data. As a result, subjective discount rates become more

Table OA.10: Model vs. Data: Asset Market and Labor Market Moments

Moment	Data	Learning Model	Rational Model
(a) Asset Market			
$Mean(pe_t)$	2.98	2.53	3.15
$\text{SD}(pe_t) \times 100$	47.4	31.2	0.0
$\text{AC}(pe_t)$	0.75	0.71	1.00
$\text{SD}(r_t) \times 100$	16.0	9.0	2.0
$\text{SD}(\mathbb{F}_t[r_{t+1}]) \times 100$	1.1	0.0	0.0
$\text{SD}(\mathbb{F}_t[\Delta e_{t+1}]) \times 100$	26.8	13.1	0.0
$\text{SD}_i(pe_{i,t}) \times 100$	22.6	15.0	0.0
$\text{SD}_i(r_{i,t}) \times 100$	5.7	4.7	2.4
$\text{SD}_i(\mathbb{F}_t[r_{i,t+1}]) \times 100$	2.6	0.1	0.0
$\text{SD}_i(\mathbb{F}_t[\Delta e_{i,t+1}]) \times 100$	14.0	11.7	0.0
(b) Labor Market			
$\text{SD}(u_t) \times 100$	2.09	1.09	0.07
$\text{AC}(u_t)$	0.91	0.83	0.98
$\text{SD}(q_t) \times 100$	8.71	5.85	0.21
$\text{AC}(q_t)$	0.94	0.92	0.98
$\text{Corr}(u_t, q_t)$	0.82	0.87	1.00
$\text{SD}_i(hl_{i,t}) \times 100$	15.70	10.20	1.13

Notes: This table compares empirical moments with model-generated moments with and without constant-gain learning. SD(·) denotes the time-series standard deviation of aggregate variables. $\text{SD}_i(\cdot)$ denotes the cross-sectional standard deviation across firms at each point in time, averaged over time. AC(·) denotes the first-order autocorrelation coefficient. Corr(·) denotes the correlation between two time series. pe_t is the log price-earnings ratio, r_t is the log stock return, Δe_t is log earnings growth, q_t is the job-filling rate, u_t is the unemployment rate, and $hl_{i,t}$ is the firm-level hiring rate. $\mathbb{F}_t[\cdot]$ denotes subjective expectations formed at time t. Data column reports empirical moments estimated from historical data. Learning model reports moments from simulations of the constant-gain learning model. Rational model reports moments from the rational expectations benchmark where agents have perfect knowledge of the earnings process.

volatile and their contribution to the variance of the job-filling rate rises. In contrast, subjective cash flow expectations lose persistence when ν is high, reducing their explanatory power for fluctuations in the job-filling rate. This contrast illustrates how fading memory can shift the relative importance of discount-rate and cash-flow channels in driving labor market outcomes.

OC Data Details

This section describes the time-series and cross-sectional data sources used in the estimation. I use quarterly data on the variables represented in the decomposition from equations (OA.35) and (OA.36): employment L_t, unemployment U_t, vacancy filling rates q_t, stock returns $r_{t,t+h}$, earnings growth $\Delta e_{t,t+h}$, price-earnings ratio pe_{t+h}, and earnings-employment ratio el_{t+h}. For each dependent variable of the decomposition, I also construct their corresponding survey expectations $\mathbb{F}_t$ and machine expectations $\mathbb{E}_t$.

Employment For realized values of employment, I first construct an annual series for the aggregate number of employees (EMP) of the S&P 500 constituents by using accounting information from the CRSP and Compustat Merged Annual Industrial Files. The data spans 1970 to 2023 and was downloaded from WRDS on May 15, 2024. I aggregate the firm-level employment data to construct a total employment series for the S&P 500. I interpolate this series to a monthly frequency by using the fitted values from real-time regressions of log annual Compustat employment series on the log monthly BLS series for total nonfarm payrolls (PAYEMS). The regressions are estimated over recursively expanding samples from an initial monthly sample that begins on 1970:01 and ends on the month of the data release for each month's total nonfarm payrolls. To ensure that the fitted values do not use future information not available on each data release, I align each monthly BLS nonfarm payroll release with the annual Compustat S&P 500 employment series from the previous calendar year. To obtain a measure of employment L_{t+1} at the beginning of period $t+1$, I convert the monthly interpolated values to a quarterly frequency by taking the value of the series as of the last month of each calendar quarter. This timing assumption ensures that the measures are consistent with the timing conventions from Section OB while still remaining known to firms by the end of period t. Data on nonfarm payrolls was downloaded through FRED on May 15, 2024.

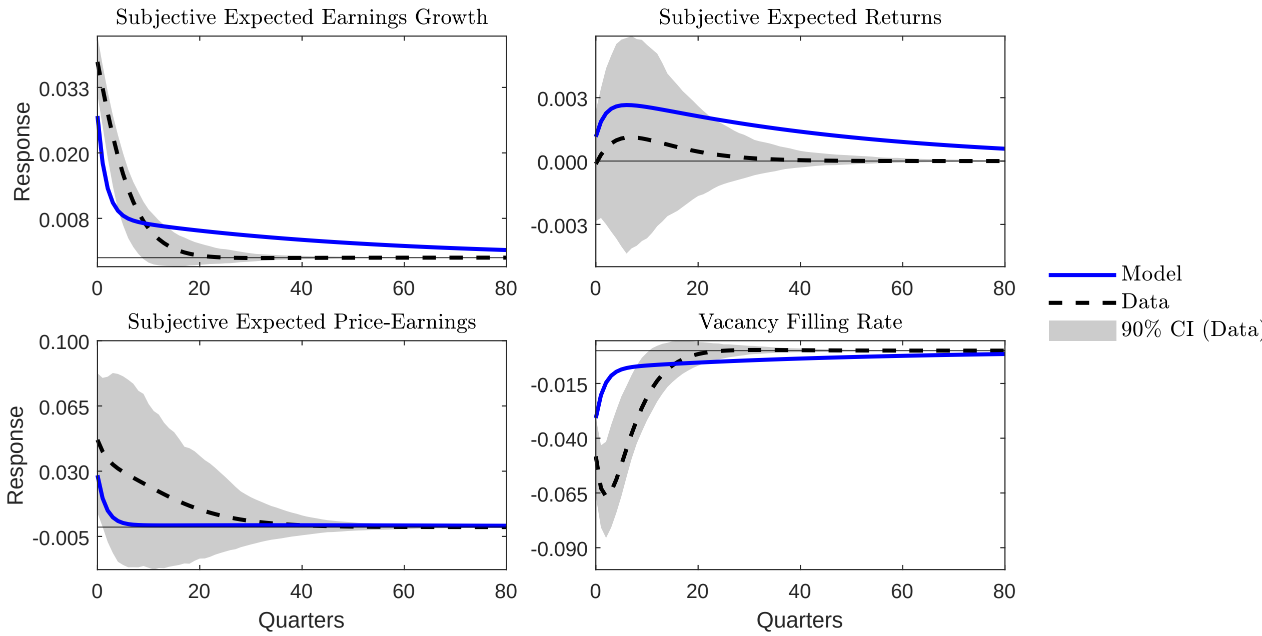


Figure OA.14: Impulse responses to a one standard deviation innovation in expected cash flow growth. Blue solid line: model-based IRFs from simulated series. Black dashed line: data-based IRFs. Shaded area: 90% bootstrap confidence interval for the data VAR. Sample: 1984Q1-2023Q4.

Vacancy Filling Rate I construct a monthly series for the number of vacancies V_t following Barnichon (2010), by using JOLTS job openings starting 2000:12 (JTS00000000JOL) and extending the series back in time using the help-wanted index before 2000:12. The vacancies data has been downloaded from available on the author's website on May 19, 2024. For realized values of unemployment U_t, I use the BLS monthly series for the unemployment level (UNEMPLOY), downloaded through FRED on May 15, 2024. Labor market tightness $\theta_t = V_t/U_t$ is the ratio between vacancies and unemployment. The job separation rate δ_t uses the corresponding series from JOLTS.

I follow Shimer (2012) in constructing the job separation rate δ_t, job finding rate f_t, and vacancy filling rate q_t. Job separation rate is the share of short-term unemployed out of total employment $\delta_t = U_t^s/L_t$, where U_t^s is the BLS series for the number of unemployed less than 5 weeks (UEMPLT5) that was downloaded through FRED on May 15, 2024. The job finding rate is:

$$f_t = 1 - \frac{U_t - U_t^s}{U_{t-1}}$$

The expression for the job finding rate follows from the unemployment accumulation equation:

$$U_t = (1 - f_t)U_{t-1} + U_t^s$$

which states that unemployment U_t consists of either the previously unemployed U_{t-1} who did not find a job $(1 - f_t)$, or the short-term unemployed U_t^s that lost a job during the current period. The vacancy filling rate is defined as the share of filled vacancies f_tV_t out of unemployment U_t:

$$q_t = \frac{f_t}{\theta_t} = \frac{f_tU_t}{V_t}$$

I first construct the vacancy filling rate q_t at the monthly frequency. To remove high-frequency fluctuations that likely reflect measurement errors, I time-aggregate the monthly series to a quarterly frequency by taking a 3-month trailing average that ends on the first month of each calendar quarter. This timing assumption ensures that the survey and machine expectations in the variance decomposition do not use advance information about vacancy filling rates that were not published at the time of each forecast. To ensure that all variables used in the variance decomposition are stationary, I follow Shimer (2012) by detrending the quarterly vacancy filling rate q_t using an HP filter with a smoothing parameter of 10^5.

Realized Stock Returns Stock market returns use monthly data on CRSP value-weighted returns including dividends (VWRETD) from the Center for Research in Security Prices (CRSP). I compute annualized log stock returns by compounding the monthly returns using $r_{t+h} \equiv \frac{1}{h}\sum_{j=1}^{12h}\log(1+VWRETD_{t+j/12})$. The data was downloaded from WRDS

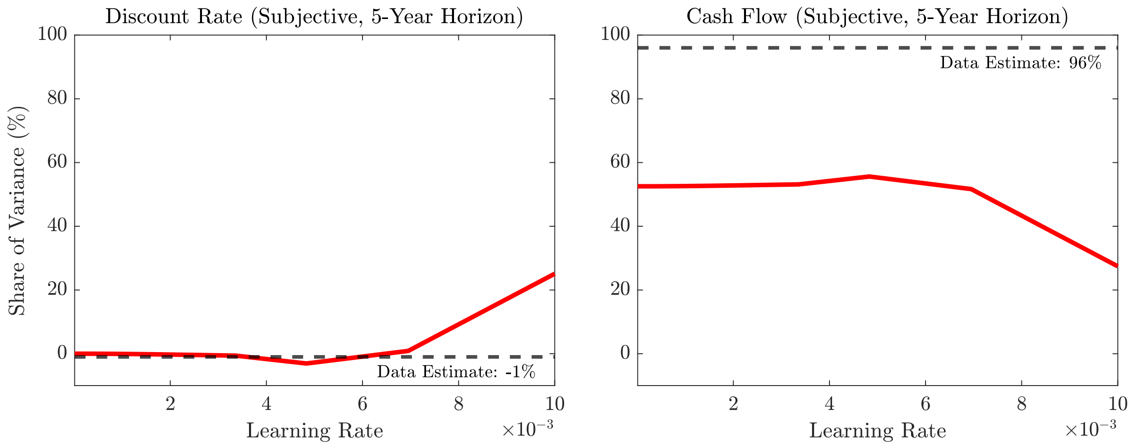


Figure OA.15: Share of job-filling rate variance explained by discount-rate (left) and cash-flow (right) components at the five-year horizon, as the constant-gain learning rate ν varies.

on May 15, 2024. When evaluating the MSE ratios of the machine relative to that of a benchmark survey, I compute machine forecasts for either annual CRSP returns or S&P 500 price growth depending on which value most closely aligns with the concept that survey respondents are asked to predict. To measure one-year stock market price growth, I use the one-year log cumulative growth rate of the S&P 500 index, $\Delta p_{t+1} \equiv \log(P_{t+1}/P_t)$. The monthly S&P index series spans the period 1957:03 to 2022:12 and was downloaded from WRDS on May 15, 2024 from the Annual Update data of the Index File on the S&P 500.

Survey Expectations of Stock Returns

CFO Survey I use survey forecasts of S&P 500 stock returns from the CFO survey to measure subjective return expectations. The CFO survey is a quarterly survey that asks respondents about their expectations for the S&P 500 return over the next 12 months and 10 years ahead, obtained from `https://www.richmondfed.org/-/media/RichmondFedOrg/research/national_economy/cfo_survey/current_historical_cfo_data.xlsx`. I use the mean point forecast for the value of the "most likely" future stock return in the estimation. More specifically, the survey asks the respondent "*over the next 12 months, I expect the average annual S&P 500 return will be: Most Likely: I expect the return to be: ___%*". The survey question for stock return expectations 10 years ahead is "*over the next 10 years, I expect the average annual S&P 500 return will be: Most Likely: I expect the return to be: ___%*". The CFO survey panel includes firms that range from small operations to Fortune 500 companies across all major industries. Respondents include chief financial officers, owner-operators, vice presidents, and directors of finance, and others with financial decision-making roles. The CFO panel has 1,600 members as of December 2022.

I take a stand on the information set of respondents when each forecast was made, and I assume that respondents could have used all data released before they completed the survey. Because the CFO survey releases quarterly forecasts at the end of each quarter, I conservatively set the response deadline for the machine forecast to be the first day of the last month of each quarter (e.g., March 1st). The data spans the periods 2001Q4 to 2023Q4 and were downloaded on March 20th, 2024. Mean point forecasts before 2020Q3 are available in column `sp_1_exp` of sheet `through_Q1_2020`; mean point forecasts from 2020Q3 and onwards are available in column `sp_12moexp_2` of sheet `CFO_SP500`. The forecast is not available in 2019Q1, 2019Q4, 2020Q1, and 2020Q2. I impute the missing forecast for 2019Q1 by linearly interpolating between the available forecasts from 2018Q4 and 2019Q2. I impute the missing forecasts for 2019Q4, 2020Q1, and 2020Q2 by interpolating with the nearest available forecast between 2019Q3 and 2020Q3. Following Nagel and Xu (2022), I assume that the forecasted S&P 500 return includes dividends and capture expectations about annualized cumulative simple net returns compounded from time t to $t+h$, i.e., $\mathbb{F}_t[R_{t,t+h}]$. To obtain survey expectations of log returns $\mathbb{F}_t[\log(1+r_{t,t+h})]$ from a survey expectation of net simple returns $\mathbb{F}_t[R_{t,t+h}]$, I use the approximation $\mathbb{F}_t[\log(1+r_{t,t+h})] \approx \log(1+\mathbb{F}_t[R_{t,t+h}])$.

To obtain long-horizon survey expectations of annualized cumulative log S&P 500 returns over the next $1 < h < 10$ years, I interpolate the forecasts across annualized 1 year and 10 year cumulative log return expectations:

$$\mathbb{F}_t[r_{t,t+h}] = \frac{10-h}{10-1}\mathbb{F}_t[r_{t,t+1}] + \frac{h-1}{10-1}\mathbb{F}_t[r_{t,t+10}], \quad h = 1, 2, \ldots, 10$$

Finally, I use the difference between cumulative long-horizon log return expectations between adjacent years (i.e., $\mathbb{F}_t[r_{t,t+h-1}]$ and $\mathbb{F}_t[r_{t,t+h}]$) to obtain $\mathbb{F}_t[r_{t+h}]$, the survey expectation of forward one-year log stock returns h years ahead:

$$\mathbb{F}_t[r_{t+h}] = h \times \mathbb{F}_t[r_{t,t+h}] - (h-1) \times \mathbb{F}_t[r_{t,t+h-1}], \quad h = 1, 2, \ldots, 10$$

IBES and Value Line I proxy expected firm-level stock returns using price growth expectations following De La O et al. (2024). Specifically, I construct expected price growth from IBES 12-month median price targets and Value Line 3–5 year median price targets, interpolating linearly for intermediate horizons.

To construct expected price growth, I combine short- and long-term price targets from two sources. For the short horizon, I use the 12-month median price targets from the Institutional Brokers Estimate System (IBES) database. For longer horizons, I use the median price targets from Value Line, which provide the expected stock price level approximately 3–5 years into the future for each firm. These targets reflect analysts' consensus expectations for each firm's stock price. I interpret the Value Line price target as the expected price level five years ahead and interpolate linearly between the IBES 12-month price target and the Value Line five-year price target to construct expected price growth for intermediate horizons between one and five years. For each firm i, expected annualized price growth over horizon h is given by:

$$\mathbb{F}_t[r_{i,t+h}] \approx \frac{1}{h} \log \left(\frac{\mathbb{F}_t[P_{i,t+h}]}{P_{i,t}} \right)$$

where $\mathbb{F}_t[P_{i,t+h}]$ is the forecasted price at horizon h, constructed through linear interpolation of IBES and Value Line targets, and $P_{i,t}$ is the observed stock price at time t. As shown in De La O et al. (2024), using price growth expectations to approximate expected firm-level stock returns is reasonably accurate, as dividends represent a relatively small component of total returns for most firms.

Gallup/UBS Survey The UBS/Gallup is a monthly survey of one-year-ahead stock market return expectations. I use the mean point forecast in our estimation and compare these to machine forecasts of the annual CRSP return. Gallup conducted 1,000 interviews of investors during the first two weeks of every month and results were reported on the last Monday of the month. The first survey was conducted on 1998:05. Until 1992:02, the survey was conducted quarterly on 1998:05, 1998:09, and 1998:11. The data on 1998:06, 1998:07, 1998:08, 1998:10, 1998:12, 1999:01, and 2006:01 are missing because the survey was not conducted on these months. I follow Adam et al. (2021) in starting the sample after 1999:02 due to missing values at the beginning of the sample.

For each month when the survey was conducted, respondents are asked about the return they expect on their own portfolio. The survey question is *"What overall rate of return do you expect to get on your portfolio in the next twelve months?"* Before 2003:05, respondents are also asked about the return they expect from an investment in the stock market during the next 12 months. The survey question is *"Thinking about the stock market more generally, what overall rate of return do you think the stock market will provide investors during the coming twelve months?"* For each month, I calculate the average expectations of returns on their own portfolio and returns on the market index. When calculating the average, survey respondents are weighted by the weight factor provided in the survey. I exclude extreme observations where a respondent reported expected returns higher than 95% or lower than -95%.

In order to construct a consistent measure of stock market return expectations over the entire sample period, I impute missing market return expectations using the fitted values from two regressions. First, I impute missing values during 1999:02-2005:12 and 2006:02-2007:10 with the fitted value from regressing expected market returns on own portfolio expectations contemporaneously, where the regression is estimated using the part of the sample where both are available. Second, I impute the one missing observation in both market and own portfolio return expectations for 2006:01 with the fitted value from regressing the market return expectations on the lagged own portfolio return expectations, where the coefficients are estimated using part of the sample where both are available, and the fitted value combines the estimated coefficients with lagged own portfolio expectations data from 2005:12. Following Nagel and Xu (2022), I assume that the forecasted stock market return includes dividends and capture expectations about annual simple net stock returns $\mathbb{F}_t[R_{t+1}]$. To obtain survey expectations of annual log returns $\mathbb{F}_t[\log(1 + r_{t+1})]$ from a survey expectation of annual net simple returns $\mathbb{F}_t[R_{t+1}]$, I use the approximation $\mathbb{F}_t[\log(1 + r_{t+1})] \approx \log(1 + \mathbb{F}_t[R_{t+1}])$. After applying all the procedures, the Gallup market return expectations series spans the periods 1999:02 to 2007:10. The data were downloaded on August 1st, 2024 from Roper iPoll: `http://ropercenter.cornell.edu/ubs-index-investor-optimism/`.

I take a stand on the information set of respondents when each forecast was made, and I assume that respondents could have used all data released before they completed the survey. Since interviews are in the first two weeks of a month (e.g., February), I conservatively set the response deadline for the machine forecast to be the first day of the survey month (e.g., February 1st), implying that I allow the machine to use information only up through the end of the previous month (e.g., through January 31st). This ensures that the machine only sees information that would have been available to all UBS/Gallup respondents for that survey month (February). This approach is conservative in the sense that it handicaps the machine, since all survey respondents who are being interviewed during the next month would have access to more timely information than the machine. Since the survey asks about the "one-year-ahead" I interpret the question to be asking about the forecast period spanning from the current survey month to the same month one year ahead.

Michigan Survey of Consumers (SOC) The SOC contains approximately 50 core questions, and a minimum of 500 interviews are conducted by telephone over the course of the entire month, each month. Table 20 of the SOC reports the probability of an increase in stock market in next year. The survey question was *"The next question is about investing in the stock market. Please think about the type of mutual fund known as a diversified stock fund. This type of mutual fund* *holds stock in many different companies engaged in a wide variety of business activities. Suppose that tomorrow someone were to invest one thousand dollars in such a mutual fund. Please think about how much money this investment would be worth one year from now. What do you think the percent chance that this one thousand dollar investment will increase in value in the year ahead, so that it is worth more than one thousand dollars one year from now?"* When using this survey forecast to compare to machine forecasts, I impute a point forecast for stock market returns using the method described in Section OC below. I compare the imputed point forecast to machine forecasts of CRSP returns.

For the SOC, interviews are conducted monthly typically over the course of an entire month. (In rare cases, interviews may commence at the end of the previous month, as in February 2018 when interviews began on January 31st 2018.) I take a stand on the information set of respondents when each forecast was made, and I assume that respondents could have used all data released before they completed the survey. Since interviews are almost always conducted over the course of an entire month (e.g., February), I conservatively set the response deadline for the machine forecast to be the first day of the survey month (e.g., February 1st), implying that I allow the machine to use information only up through the end of the previous month (e.g., through January 31st). This ensures that the machine only sees information that would have been available to all respondents for that survey month (February). This approach is conservative in the sense that it handicaps the machine, since all survey respondents who are being interviewed during the next month would have access to more timely information than the machine. Since the survey asks about the "year ahead" I interpret the question to be asking about the forecast period spanning the period running from the current survey month to the same month one year ahead. The data spans 2002:06 to 2023:12. The SOC responses were obtained from `https://data.sca.isr.umich.edu/data-archive/mine.php` and downloaded on May 15, 2024.

Livingston Survey Stock Price Forecast I obtain the Livingston Survey S&P 500 index forecast (SPIF) from the Federal Reserve Bank of Philadelphia, and use the mean values in our structural and forecasting models. I compare the one-year growth in these forecasts to machine forecasts of S&P 500 price growth. Our sample spans 1947:06 to 2023:06. The forecast series were downloaded on January 24, 2024.

The survey provides semi-annual forecasts on the level of the S&P 500 index. Participants are asked to provide forecasts for the level of the S&P 500 index for the end of the current survey month, 6 months ahead, and 12 months ahead. I use the mean of the respondents' forecasts each period, where the sample is based on about 50 observations. Most of the survey participants are professional forecasters with "formal and advanced training in economic theory and forecasting and use econometric models to generate their forecasts." Participants receive questionnaires for the survey in May and November, after the Consumer Price Index (CPI) data release for the previous month. All forecasts are typically submitted by the end of the respective month of May and November. The results of the survey are released near the end of the following month, on June and December of each calendar year. The exact release dates are available on the Philadelphia Fed website, at the header of each news release. I take a stand on the information set of the respondents when each forecast was made by assuming that respondents could have used all data released before they completed the survey. Since all forecasts are typically submitted by the end of May and November of each calendar year, I set the response deadline for the machine forecast to be the first day of the last month of June and December, implying that I allow the machine to use information only up through the end of the May and November.

I follow Nagel and Xu (2021) in constructing one-year stock price growth expectations from the level forecasts. Starting from June 1992, I use the ratio between the 12-month level forecast ($\texttt{SPIF\_12M}_t$) and 0-month level nowcasts ($\texttt{SPIF\_ZM}_t$) of the S&P 500 index. Before June 1992, the 0-month nowcast is not available. Therefore I use the annualized ratio between the 12-month ($\texttt{spi12}_t$) and 6-month ($\texttt{spi6}_t$) level forecast of the S&P 500 index

$$\mathbb{F}_t^{(Liv)}\left[\frac{P_{t+1}}{P_t}\right] \approx \begin{cases} \frac{\mathbb{F}_t^{(Liv)}[P_{t+1}]}{\mathbb{F}_t^{(Liv)}[P_t]} = \frac{\texttt{SPIF\_12M}_t}{\texttt{SPIF\_ZM}_t} & \text{if } t \geq 1992M6 \\ \left(\frac{\mathbb{F}_t^{(Liv)}[P_{t+1}]}{\mathbb{F}_t^{(Liv)}[P_{t+6}]}\right)^2 = \left(\frac{\texttt{spi12}_t}{\texttt{spi6}_t}\right)^2 & \text{if } t < 1992M6 \end{cases}$$

where P_t is the S&P 500 index and t indexes the survey's response deadline. To obtain a survey expectation of the log change in price growth I use the approximation $\mathbb{F}_t(\Delta p_{t+1}) \approx \log(\mathbb{F}_t[P_{t+1}]) - \log(P_t)$.

Conference Board (CB) Survey Respondents provide the categorical belief of whether they expect stock prices to "increase," "decrease," or stay the "same" over the next year. Since the survey asks respondents about stock prices in the "year ahead," I interpret the question to be asking about the forecast period from the end of the current survey month to the end of the same month one year ahead. When we use this qualitative survey forecast to compare to machine forecasts, we impute a point forecast for stock market returns using the method described in Section OC below. I compare the imputed point forecast to machine forecasts of CRSP returns.

The survey is conducted monthly and I use the survey responses over 1987:04 to 2022:08. The data was downloaded on September 26, 2022. The survey uses an address-based mail sample design. Questionnaires are mailed to households on or about the first of each month. Survey responses flow in throughout the collection period, with the sample close-out for preliminary estimates occurring around the 18th of the month. Any responses received after then are used to produce final estimates for the month, which are published with the following month's data. Conversations with those knowledgeable about the survey suggested that most panelists respond early. Any responses received after around the 20th of the month–regardless of when they are filled out–are included in the final (but not preliminary) numbers.

I take a stand on the information set of the respondents when each forecast was made by assuming that respondents could have used all data released before they completed the survey. Since questionnaires reach households on or about the first of each month (e.g., February 1st) and most respondents respond early, I conservatively set the response deadline for the machine forecast to be the first day of the survey month (e.g., February 1st), implying that I allow the machine to use information only up through the end of the previous month (e.g., January 31st).

Converting Qualitative Forecasts to Point Forecasts (SOC and CB) I use the SOC probability to impute a quantitative point forecast of stock returns using a linear regression of CFO point forecasts for returns onto the SOC probablity of a price increase. The SOC asks respondents about the percent chance that an investment will "increase in value in the year ahead." I interpret this as asking about the ex dividend value, i.e., about price price growth. The CFO survey is conducted quarterly, where the survey quarters span 2001Q4 to 2021Q1. The SOC survey is conducted monthly, where survey months span 2002:06 to 2021:12. Since the CFO is a quarterly survey, the regression is estimated in real-time over a quarterly overlapping sample. Since the CFO survey is conducted during the last month of the quarter while the SOC is conducted monthly, I align the survey months between CFO and SOC by regressing the quarterly CFO survey point forecast with the qualitative SOC survey response during the last month of the quarter.

Since the SOC survey question is interpreted as asking about S&P 500 price growth while the CFO survey question asks about stock returns including dividends, I follow Nagel and Xu (2021) in subtracting the current dividend yield of the CRSP value weighted index from the CFO variable before running the regression. After estimating the regression, I then add back the dividend yield to the fitted value to obtain an imputed SOC point forecast of stock returns including dividends. Specifically, at time t, I assume that the CFO forecast of stock returns, $\mathbb{F}_t^{\mathrm{CFO}}[r_{t,t+1}]$, minus the current dividend yield, D_t/P_t, is related to the contemporaneous SOC probability of an increase in the stock market next year, $P_{t,t+1}^{\mathrm{SOC}}$, by:

$$\mathbb{F}_t^{\mathrm{CFO}}[r_{t,t+1}] - D_t/P_t = \beta_0 + \beta_1 P_{t,t+1}^{\mathrm{SOC}} + \epsilon_t.$$

The final imputed SOC point forecast is constructed as $\mathbb{F}_t^{\mathrm{SOC}}[r_{t,t+1}] = \hat{\beta}_0 + \hat{\beta}_1 P_{t,t+1}^{\mathrm{SOC}} + D_t/P_t$. I first estimate the coefficients of the above regression over an initial overlapping sample of 2002Q2 to 2004Q4, where the quarterly observations from the CFO survey is regressed on the SOC survey responses from the last month of each calendar quarter. Using the estimated coefficients and the SOC probability from 2005:03 gives us the point forecast of the one-year stock return from 2005Q1 to 2006Q1. I then re-estimate this equation, recursively, adding one quarterly observation to the end of the sample at a time, and storing the fitted values. This results in a time series of SOC point forecasts $\mathbb{F}_t^{\mathrm{SOC}}[r_{t,t+1}]$ spanning 2005Q1 to 2021Q1.

The same procedure is done for the Conference Board Survey, except I replace $P_{t,t+1}^{\mathrm{SOC}}$ by $P_{t,t+1}^{\mathrm{CB}}$, a ratio of the proportion of those who respond with "increase" to the sum of "decrease" and "same." The CB survey asks respondents to provide the categorical belief of whether they expect stock prices to "increase," "decrease," or stay the "same" over the next year. I interpret this as asking about price price growth. Since the CB survey question is interpreted as asking about S&P 500 price growth while the CFO survey question asks about stock returns including dividends, I follow Nagel and Xu (2021) in subtracting the current dividend yield of the CRSP value weighted index from the CFO variable before running the regression. After estimating the regression, I then add back the dividend yield to the fitted value to obtain an imputed CB point forecast of stock returns including dividends.

The CFO survey is conducted quarterly, where the survey quarters span 2001Q4 to 2021Q1. The CB survey is conducted monthly, where survey months span 1987:04 to 2022:08. The regression is first estimated over an initial overlapping sample of 2001Q4 to 2004Q4, where the quarterly observations from the CFO survey is regressed on the CB survey responses from the last month of each calendar quarter. Using the estimated coefficients and the CB survey response $P_{t,t+1}^{\mathrm{CB}}$ from 2005:03 gives us the point forecast of the stock return from 2005Q1 to 2006Q1. I then re-estimate this equation, recursively, adding one observation to the end of the sample at a time, and storing the fitted values. This results in a time series of CB point forecasts $\mathbb{F}_t^{\mathrm{CB}}[r_{t,t+1}]$ over 2005Q1 to 2021Q1.

Nagel and Xu Individual Investor Expectations Nagel and Xu (2021)'s individual investor expectations series for returns covers 1972-1977 (Annual) and 1987Q2-2023Q4 (Quarterly) and combine data from the following surveys:

1. UBS/Gallup: 1998:06-2007:10; Survey captures respondents' expected stock market returns, in percent, over a 1-year horizon.

2. Michigan Survey of Consumers (SOC): 2002:04-2023:12; Respondents provide the probability of a rise in the stock market over a 1-year horizon.

3. Conference Board (CB): 1987:04-2022:08; Respondents provide the categorial opinion whether they expect stock prices to rise, or stay about where they are, or decline over the next year.

4. Vanguard Research Initiative (VRI): 2014:08; Survey captures respondents' expected stock market returns, in percent, over a 1-year horizon.

5. Roper: 1974-1977, annual, observed June of each calendar year; Respondents provide the categorial opinion whether they expect stock prices to rise, or stay about where they are, or decline over the next year.

6. Lease, Lewellen, and Schlarbaum (1974, 1977): 1972-1973, annual, observed July of each calendar year; Survey captures respondents' expected stock market returns, in percent, over a 1-year horizon.

Among these sources, UBS/Gallup and VRI provide direct, point forecasts of expected stock returns, while SOC, CB, and Roper offer qualitative or probabilistic information that requires conversion to consistent return expectations. Nagel and Xu (2021) construct their final series using the following procedure:

1. Start with UBS/Gallup for 1998:06-2007:10 and VRI for 2014:08 since they capture the respondents' expected stock returns relatively closely (other surveys only provide qualitative measures).

2. Regress SOC on UBS/Gallup and VRI using periods of overlapping coverage (2002:04-2007:10). Use the fitted values from this regression to impute missing data for 2007:11-2023:12 (excluding 2014:08).

3. Regress CB on UBS/Gallup and VRI using periods of overlapping coverage (1998:06-2007:10). Use the fitted values from this regression to impute missing data for 1987:04-1998:05 (using CB) and 1974-1977 (using Roper).

4. Use the coefficients from regressing CB on UBS/Gallup and VRI (from step 3) to compute fitted values that convert the probabilistic forecast from Roper into point forecasts of stock returns.

5. Convert expected returns to expected excess returns by subtracting the average 1-year Treasury yield measured at the beginning of the survey month.

6. Aggregate monthly series to a quarterly frequency by taking the average expectation within calendar quarters.

Risk-Free Rates

Realized Risk-Free Rates As a measure of realized risk-free rates r_t^f, I obtain daily series for the annualized three-month Treasury bill rate (DTB3), downloaded from FRED on May 15, 2024. To match the definition used as the target variable in the Survey of Professional Forecasters (SPF), I time-aggregate the daily realized risk-free rate series to a quarterly frequency by taking the quarterly average, as discussed below.

Survey Expectations of Risk-Free Rates I obtain subjective expectations about risk-free rates from median forecasts for the annualized three-month Treasury bill rate from the Survey of Professional Forecasters (SPF). The SPF provides forecasts at the one and ten year horizons. For one year ahead forecasts (TBILL), respondents are asked to provide quarterly forecasts of the quarterly average three-month Treasury bill rate, in percentage points, where the forecasts are for the quarterly average of the underlying daily levels. I interpret the survey to be asking about annual net simple rates $\mathbb{F}_t[R_{t,t+1}^f]$, and approximate the expected log risk-free rate as $\mathbb{F}_t[r_{t,t+1}^f] \approx \log(1 + \mathbb{F}_t[R_{t,t+1}^f])$. For ten year ahead forecasts (BILL10), respondents are asked to provide forecasts for the annual-average rate of return to three-month Treasury bills over the next 10 years, in percentage points. The ten year ahead forecasts are available only for surveys conducted in the first quarter of each calendar year. I interpret the survey to be asking about annualized cumulative net simple rates compounded from the survey quarter to the same quarter that is ten years after the survey year $\mathbb{F}_t[R_{t,t+10}^f]$, and approximate the expected log risk-free rate as $\mathbb{F}_t[r_{t,t+10}^f] \approx \log(1+\mathbb{F}_t[R_{t,t+10}^f])$. To obtain long-horizon survey expectations of annualized log three-month Treasury bill rates over the next $1 < h < 10$ years, I interpolate the forecasts across annualized 1 year and 10 year return expectations:

$$\mathbb{F}_t[r_{t,t+h}^f] = \frac{10-h}{10-1}\mathbb{F}_t[r_{t,t+1}^f] + \frac{h-1}{10-1}\mathbb{F}_t[r_{t,t+10}^f], \quad h = 1, 2, \ldots, 10$$

Finally, I use the difference between the cumulative annualized long-horizon log three-month Treasury bill rate expectations between adjacent years (i.e., $\mathbb{F}_t[r_{t,t+h-1}^f]$ and $\mathbb{F}_t[r_{t,t+h}^f]$) to obtain $\mathbb{F}_t[r_{t+h}^f]$, the time t survey expectation of annualized forward log three-month Treasury bill rate h years ahead:

$$\mathbb{F}_t[r_{t+h}^f] = h \times \mathbb{F}_t[r_{t,t+h}^f] - (h-1) \times \mathbb{F}_t[r_{t,t+h-1}^f], \quad h = 1, 2, \ldots, 10$$

The surveys are sent out at the end of the first month of each quarter, and collected in the second or third week of the middle month of each quarter. When constructing machine learning forecasts for the risk-free rate, I assume that forecasters could have used all data released before the survey deadlines for the SPF, which are posted online at the Federal Reserve Bank of Philadelphia website. Since surveys are typically sent out at the end of the first month of each quarter, I make the conservative assumption that respondents only had data released by the first day of the second month of each quarter.

Realized Earnings I use IBES street earnings per share (EPS) data that start in 1983:Q4 as the forecast target for IBES analysts. Following the recommendation of Hillenbrand and McCarthy (2024), I use Street earnings as the forecast target for IBES analysts. Street earnings differ from GAAP earnings by excluding discontinued operations, extraordinary charges, and other non-operating items. According to the IBES user guide, analysts submit forecasts after backing out these transitory components, and IBES constructs the realized series to align with those forecasts. While analysts have some discretion over which items to exclude, Hillenbrand and McCarthy (2024) demonstrate that the target of these forecasts corresponds closely to earnings before special items in Compustat, suggesting that street earnings accurately reflect the measure analysts are targeting. To convert EPS to total earnings, I multiply the resulting quarterly EPS series by the quarterly S&P 500 divisor, available at: `https://ycharts.com/indicators/sp_500_divisor`. The final quarterly total earnings series spans the period 1983:Q4 to 2023:Q4. To extend the sample back to 1965Q1, I use quarterly Compustat data on earnings before special items. As noted in Hillenbrand and McCarthy (2024), this measure closely tracks IBES street earnings, indicating it accurately reflects analysts' forecast targets. IBES street earnings data and Compustat data has been downloaded from WRDS on July 19, 2025. The divisor data were downloaded on July 21, 2025.

Survey Expectations of Earnings I obtain monthly survey data for the median analyst earnings per share forecast and actual earnings per share from the Institutional Brokers Estimate System (IBES) via Wharton Research Data Services (WRDS). The data spans the period 1976:01 to 2023:12.

Short-Term Growth (STG) Expectations I build measures of aggregate S&P 500 earnings expectations growth using the constituents of the S&P 500 at each point in time following De La O and Myers (2021). I first construct expected earnings expectations for aggregate earnings h-months-ahead as:

$$\mathbb{F}_t[E_{t+h}] = \Omega_t \sum_{i \in x_t} \mathbb{F}_t[EPS_{i,t+h}] \cdot S_{i,t}/Divisor_t$$

where $\mathbb{F}$ is the median analyst survey forecast, E is aggregate S&P 500 earnings, EPS_i is earning per share of firm i among all S&P 500 firms x_t for which I have forecasts in IBES for $t+h$, S_i is shares outstanding of firm i, and $Divisor_t$ is calculated as the S&P 500 market capitalization divided by the S&P 500 index. I obtain the number of outstanding shares for all companies in the S&P500 from Compustat. IBES estimates are available for most but not all S&P 500 companies. Following De La O and Myers (2021), I multiply this aggregate by $\mathit{\Omega}_t$, a ratio of total S&P 500 market value to the market value of the forecasted companies at t to account for the fact that IBES does not provide earnings forecasts for all firms in the S&P 500 in every period.

IBES database contains earning forecasts up to five annual fiscal periods (FY1 to FY5) and as a result, I interpolate across the different horizons to obtain the expectation over the next 12 months. This procedure has been used in the literature, including De La O and Myers (2021). Specifically, if the fiscal year of firm XYZ ends nine months after the survey date, I have a 9-month earning forecast $\mathbb{F}_t[E_{t+9}]$ from FY1 and a 21-month forecast $\mathbb{F}_t[E_{t+21}]$ from FY2. I then obtain the 12-month ahead forecast by interpolating these two forecasts as follows,

$$\mathbb{F}_t[E_{t+12}] = \frac{9}{12}\mathbb{F}_t[E_{t+9}] + \frac{3}{12}\mathbb{F}_t[E_{t+21}].$$

To convert the monthly forecast to quarterly frequency, I use the forecast made in the middle month of each quarter, and construct one-year earnings expectations from 1976Q1 to 2023Q4 and the earning expectation growth is calculated as an approximation following following De La O and Myers (2021):

$$\mathbb{F}_t\left(\Delta e_{t+12}\right) \approx \ln\left(\mathbb{F}_t[E_{t+12}]\right) - e_t$$

where e_t is log earnings for S&P 500 at time t calculated as $e_t = \log\left(EPS_t \cdot Divisor_t\right)$, where EPS_t is the earnings per share for the S&P 500 obtained from Shiller's data depository and S&P Global, as described above.

Long-Term Growth (LTG) Expectations I construct long term expected earnings growth (LTG) for the S&P 500 following Bordalo et al. (2019). Specifically, I obtain the median firm-level LTG forecast from IBES, and aggregate the value-weighted firm-level forecasts,

$$LTG_t = \sum_{i=1}^{S} LTG_{i,t} \frac{P_{i,t}Q_{i,t}}{\sum_{i=1}^{S} P_{i,t}Q_{i,t}}$$

where S is the number of firms in the S&P 500 index, and where $P_{i,t}$ and $Q_{i,t}$ are the stock price and the number of shares outstanding of firm i at time t, respectively. $LTG_{i,t}$ is the median forecast of firm i's long term expected earnings growth. The data spans the periods from 1981:12 to 2023:12. All data were downloaded in July 19, 2025.

Finally, I use the difference between survey expectations of log earnings between adjacent years (i.e., $\mathbb{F}_t[e_{t+h-1}]$ and $\mathbb{F}_t[e_{t+h}]$) to obtain $\mathbb{F}_t[\Delta e_{t+h}] = \mathbb{F}_t[e_{t+h}] - \mathbb{F}_t[e_{t+h-1}]$, the time t survey expectation of forward one-year log earnings growth

$h = 1, 2$ years ahead. For the $h = 3, 4, 5$ year horizon, I interpret the IBES's Long-Term Growth (LTG) forecast as the forward annual log earnings growth:

$$\mathbb{F}_t[\Delta e_{t+h}] = \begin{cases} \mathbb{F}_t[e_{t+h}] - \mathbb{F}_t[e_{t+h-1}] & \text{if } h = 1, 2 \text{ years} \\ LTG_t & \text{if } h = 3, 4, 5 \text{ years} \end{cases}$$

To estimate any biases in IBES analyst forecasts, the dynamic machine algorithm takes as an input a likely date corresponding to information analysts could have known at the time of their forecast. IBES does not provide an explicit deadline for their forecasts to be returned. Therefore I instead use the "statistical period" day (the day when the set of summary statistics was calculated) as a proxy for the deadline. I set the machine deadline to be the day before this date. The statistical period date is typically between day 14 and day 20 of a given month, implying that the machine deadline varies from month to month. As the machine learning algorithm uses mixed-frequency techniques adapted to quarterly sampling intervals, while the IBES forecasts are monthly, I compare machine and IBES analyst forecasts as of the middle month of each quarter, considering 12-month ahead forecast from the beginning of the month following the survey month.

Price-Earnings Ratio I construct a quarterly series for the price-earnings ratio $PE_t \equiv P_t/E_t$ using the end-of-quarter S&P 500 stock price index P_t and the S&P 500 quarterly total earnings E_t. I infer subjective expectations of the log price-earnings ratio $\mathbb{F}_t[pe_{t+h}]$ by combining the current log price-earnings ratio pe_t with h year ahead subjective expectations of annual log stock returns $\mathbb{F}_t[r_{t+h}]$ and annual log earnings growth $\mathbb{F}_t[\Delta e_{t+h}]$, following the approach used in De La O and Myers (2021). Rearrange the Campbell and Shiller (1988) present value identity for the price-earnings ratio in equation (OA.32) to express the future log price-earnings ratio as a function of current log price-earnings, log earnings growth, and log stock returns:

$$pe_{t+h} = \frac{1}{\rho^h} pe_t - \frac{1}{\rho^h} \sum_{j=1}^{h} \rho^{j-1} (c_{pe} + \Delta e_{t+j} - r_{t+j})$$

where the equation holds both ex-ante and ex-post. Apply subjective expectations $\mathbb{F}_t$ on both sides of the equation:

$$\mathbb{F}_t[pe_{t+h}] = \frac{1}{\rho^h} pe_t - \frac{1}{\rho^h} \sum_{j=1}^{h} \rho^{j-1} (c_{pe} + \underbrace{\mathbb{F}_t[\Delta e_{t+j}]}_{\text{Survey (IBES)}} - \underbrace{\mathbb{F}_t[r_{t+j}]}_{\text{Survey (CFO)}}) \tag{OA.174}$$

where subjective expectations about j years ahead forward annual log stock returns $\mathbb{F}_t[r_{t+j}]$ and forward annual log earnings growth $\mathbb{F}_t[\Delta e_{t+j}]$ use survey forecasts from the CFO survey and IBES, respectively. I construct firm-level price-earnings expectations by applying the same log-linear approximation to firm-level expectations of stock returns (from IBES and Value Line) and earnings growth (from IBES).

Earnings-Employment Ratio The current earnings-employment ratio is defined as $EL_t \equiv E_t/L_{t+1}$, where E_t denotes quarterly total earnings for the S&P 500 and L_{t+1} is the employment stock at the beginning of period $t + 1$. I measure L_{t+1} using end-of-period employment levels within each quarter. This timing assumption ensures that the measures are consistent with the timing conventions from Section OB while still remaining known to firms by the end of period t.

Machine Learning Forecasts For each survey forecast, I also construct their corresponding machine learning forecast by estimating a Long Short-Term Memory (LSTM) neural network:

$$\mathbb{E}_t[y_{t+h}] = G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$$

where y_{t+h} denotes the variable y to be predicted h years ahead of time t, and $\mathcal{X}_t$ is a large input dataset of right-hand-side variables including the intercept. $G(\mathcal{X}_t, \boldsymbol{\beta}_{h,t})$ denotes predicted values from a LSTM neural network that can be represented by a (potentially) high-dimensional set of finite-valued parameters $\boldsymbol{\beta}_{h,t}$. The machine learning model is estimated using an algorithm that takes into account the data-rich environment in which firms operate in (Bianchi et al., 2022 and Bianchi et al., 2024). When constructing machine learning forecasts of each variable, I allow the machine to use only information that would have been available to all survey respondents at the time of each forecast. See Section OE for details about the machine learning algorithm and predictor variables. Machine expectations about the price-earnings ratio $\mathbb{E}_t[pe_{t+h}]$ is constructed similarly to the survey counterpart, by replacing the survey forecasts of stock returns and earnings growth on the right-hand side of equation (OA.174) with the corresponding machine learning forecasts.

For the cross-sectional decomposition, I construct analogous machine learning forecasts of returns, earnings growth, and price-earnings ratios at the firm level using the same LSTM framework, applied to portfolio-specific predictors and outcomes. To keep the machine learning algorithm tractable, I re-estimate the model parameters and update the hyper-parameter cross-validation every four quarters.

OD Method of Simulated Moments Estimation

This section describes the Method of Simulated Moments (MSM) implementation for the constant-gain learning model. The estimation proceeds in three steps: (i) define the parameters to be estimated, (ii) specify the empirical statistics S_N to be matched, and (iii) derive the model-implied counterparts $S(\theta)$ that map parameters into moments.

Parameters estimated. The MSM estimation targets a parameter vector

$$\theta \equiv (\nu, \phi, \sigma_u, r_f, \gamma, \phi_e, \sigma_v),$$

where ν is the constant-gain learning rate, (ϕ, σ_u) govern the aggregate earnings process, r_f is the risk-free rate, γ is relative risk aversion, and (ϕ_e, σ_v) govern the idiosyncratic earnings process. Other parameters (e.g., separation rate, matching-function elasticity) are calibrated externally as described in Table 1.

Empirical statistics. The set of empirical statistics S_N used in the objective function consists of twelve moments: the volatility and autocorrelation of aggregate price–earnings ratios, the volatility of aggregate stock returns, the volatility of aggregate earnings growth, the volatility of the vacancy filling rate, three statistics for idiosyncratic earnings growth (variance, autocorrelation, and cross-sectional dispersion), the volatility of idiosyncratic stock returns, the volatility of idiosyncratic price–earnings ratios, the mean price–earnings ratio, and the Coibion–Gorodnichenko regression slopes at horizons $h = 4$ and $h = 8$. The degrees of freedom for the over-identification test therefore correspond to twelve matched moments and seven free parameters.

Model mappings. The model delivers simulated analogs $S(\theta)$ for each of the nine empirical statistics. These mappings tie the data moments directly to the estimated parameters. For the earnings growth block, aggregate earnings follow the process

$$e_t = \mu + \phi e_{t-1} + u_t, \qquad u_t \sim \mathcal{N}(0, \sigma_u^2),$$

so that $\Delta e_t = (\phi - 1)e_{t-1} + u_t$. In the stationary distribution, the variance of earnings is $\text{Var}(e_t) = \sigma_u^2/(1 - \phi^2)$. This leads to exact mappings for the variance and autocorrelation of growth:

$$\text{Var}(\Delta e_t) = \frac{(\phi-1)^2}{1-\phi^2}\sigma_u^2 + \sigma_u^2, \tag{OA.175}$$

$$\rho_{\Delta e}(1) = \frac{(\phi-1)^2 \frac{\phi}{1-\phi^2}\sigma_u^2 + (\phi-1)\sigma_u^2}{\frac{(\phi-1)^2}{1-\phi^2}\sigma_u^2 + \sigma_u^2}. \tag{OA.176}$$

These moments provide direct information about the persistence and volatility parameters (ϕ, σ_u). For expected returns, strip prices are given by

$$P_t^{(h)} = \exp\{A(h) + B(h)\mathbb{F}_t[\mu] + \phi^h e_t\},$$

where the coefficients are defined recursively as

$$A(h) = A(h-1) - r_f + \tfrac{1}{2}C(h)\big(C(h) - 2\gamma\big)\sigma_u^2, \tag{OA.177}$$

$$B(h) = \frac{1-\phi^h}{1-\phi}, \tag{OA.178}$$

$$C(h) = \nu B(h-1) + \phi^{h-1}. \tag{OA.179}$$

The expected return on a strip of maturity h is then

$$\mathbb{F}_t[R_{t+1}^{(h)}] = \exp\{r_f + C(h)\gamma\sigma_u^2\}. \tag{OA.180}$$

The aggregate stock return is constructed as the value-weighted average of strip returns,

$$R_{t+1} = \sum_{h\geq 1} w_{t,h} R_{t+1}^{(h)}, \qquad w_{t,h} = \frac{P_t^{(h)}}{\sum_{k\geq 1} P_t^{(k)}}. \tag{OA.181}$$

Simulation of (OA.180)-(OA.181) yields the model-implied mean and volatility of returns, and thereby helps to identify γ jointly with (ϕ, σ_u). For the price-earnings ratio, the Campbell-Shiller log-linearization implies

$$pe_t = c_{pe} - r_{t+1} + \Delta e_{t+1} + \rho pe_{t+1}, \tag{OA.182}$$

Simulating (OA.182) provides the model-implied volatility and persistence of pe_t, which are jointly informative about ν, ϕ, and γ. For the Coibion and Gorodnichenko (2015) regression coefficient, start by noting that earnings follow an AR(1) process

$$e_t = \mu + \phi e_{t-1} + u_t, \qquad u_t \sim \mathcal{N}(0, \sigma_u^2),$$

so that

$$\Delta e_t = (\phi - 1)e_{t-1} + u_t.$$

Beliefs update with constant gain ν according to

$$\mathbb{F}_t[\mu] - \mathbb{F}_{t-1}[\mu] = \nu\big(\Delta e_t - \mathbb{F}_{t-1}[\Delta e_t]\big).$$

The one-step forecast error is

$$\mathrm{FE}_{t,1} = \Delta e_t - \mathbb{F}_{t-1}[\Delta e_t] = u_t - \mathbb{F}_{t-1}[\mu],$$

and the h-step forecast revision is

$$\mathrm{Rev}_{t,h} = \mathbb{F}_t[\Delta e_{t+h}] - \mathbb{F}_{t-1}[\Delta e_{t+h}] = \phi^{h-1}\nu\,\mathrm{FE}_{t,1} + \phi^{h-1}(\phi - 1)\Delta e_t.$$

Iterating the updating recursion gives

$$\mathbb{F}_t[\mu] = \nu \sum_{j=0}^{\infty} (1-\nu)^j u_{t-j},$$

which implies

$$\mathrm{Var}(\mathbb{F}_t[\mu]) = \tfrac{\nu}{2-\nu}\sigma_u^2, \qquad \mathrm{Var}(\mathrm{FE}_{t,1}) = \tfrac{2}{2-\nu}\sigma_u^2.$$

The covariance between earnings growth and the forecast error is

$$\mathrm{Cov}(\Delta e_t, \mathrm{FE}_{t,1}) = \sigma_u^2 - (\phi - 1)\,\frac{\nu}{1-\phi(1-\nu)}\sigma_u^2,$$

so that

$$\frac{\mathrm{Cov}(\Delta e_t, \mathrm{FE}_{t,1})}{\mathrm{Var}(\mathrm{FE}_{t,1})} = \frac{2-\nu}{2} \cdot \frac{1-\phi+\nu}{1-\phi+\phi\nu}.$$

Therefore the CG slope is

$$\beta^{CG}(h) = \frac{\mathrm{Cov}(\mathrm{Rev}_{t,h}, \mathrm{FE}_{t,1})}{\mathrm{Var}(\mathrm{FE}_{t,1})} = \phi^{h-1}\left[\nu + (\phi - 1)\,\frac{2-\nu}{2} \cdot \frac{1-\phi+\nu}{1-\phi+\phi\nu}\right].$$

These statistics discipline the constant-gain parameter ν.

MSM criterion. The estimator minimizes the distance between empirical and model-implied moments:

$$\hat{\theta}_N = \arg\min_{\theta}\,(S_N - S(\theta))' W_N^{-1}(S_N - S(\theta)).$$

In the first step, the weighting matrix W_N is set to the identity. In the second step, it is replaced by a heteroskedasticity and autocorrelation robust covariance matrix of the empirical moments, with regression-based moments adjusted by the delta method. The minimized criterion also yields a test of overidentifying restrictions with degrees of freedom equal to the number of moments minus the number of estimated parameters.

Estimation results. Table OA.11 reports the results of the MSM estimation. Panel A compares each data moment to its model counterpart and reports the t-statistic of the difference based on the step two weighting matrix. Panel B lists the estimated values of the parameters ν, ϕ, σ_u, r_f, and γ. Panel C reports the parameters that are held fixed during estimation, such as the time discount factor ρ, matching efficiency B, matching elasticity η, separation rate δ, and vacancy posting cost κ. The final rows of Panel B report the minimized value of the MSM criterion and the associated p-value of the overidentification test. These results summarize how the model parameters map into the observed dynamics of earnings, returns, price-earnings ratios, and learning coefficients, providing a joint test of the model's ability to replicate the empirical moments.

Table OA.11: Model Estimation Outcome

Moment or parameter	Data	Model	t statistic
Panel A: Moments			
Mean log stock return	0.072	0.088	-0.510
SD log stock return	0.160	0.118	0.568
Mean log risk free rate	0.046	0.045	0.144
Mean of log price earnings	2.980	2.392	0.424
SD of log price earnings	0.285	0.293	-0.084
AC of log price earnings	0.750	0.798	-0.457
SD of aggregate earnings growth	0.268	0.294	-0.455
AC of aggregate earnings growth	-0.144	-0.142	-0.045
SD of idiosyncratic earnings growth	0.112	0.091	0.388
AC of idiosyncratic earnings growth	-0.027	-0.023	-0.304
CG slope $h = 4$ aggregate	-0.263	-0.266	0.063
CG slope $h = 8$ aggregate	-0.463	-0.454	-0.040
Panel B: Estimated Parameters			
Gain coefficient ν		0.013	
AR coefficient aggregate ϕ		0.854	
AR coefficient idiosyncratic ϕ_e		0.936	
Aggregate shock standard deviation σ_u		0.271	
Idiosyncratic shock standard deviation σ_v		0.086	
Risk free rate r_f		0.045	
Risk aversion γ		1.647	
Test statistic W_N		728.457	
p value of W_N		0.000	
Panel C: Assigned Parameters			
Time discount factor ρ (Campbell and Shiller (1988))		0.980	
Matching function efficiency B (Kehoe et al. (2023))		0.562	
Matching function elasticity η (Kehoe et al. (2023))		0.500	
Separation rate δ (Kehoe et al. (2023))		0.286	
Per worker hiring cost κ (Elsby and Michaels (2013))		0.314	

Notes: This table reports data moments, moments from the estimated model, parameter estimates, and test statistics. The model is calibrated at an annual frequency.

OE Machine Learning

OE.1 Machine Algorithm Details

The basic dynamic algorithm follows the six step approach of Bianchi et al. (2022) of 1. Sample partitioning, 2. In-sample estimation, 3. Training and cross-validation, 4. Grid reoptimization, 5. Out-of-sample prediction, and 6. Roll forward and repeat. We refer the interested reader to that paper for details and discuss details of the implementation here only insofar as they differ. At time t, a prior sample of size $\dot{T}$ is partitioned into two subsample windows: a *training sample* consisting of the first T_E observations, and a hold-out *validation sample* of T_V subsequent observations so that $\dot{T} = T_E + T_V$. The training sample is used to estimate the model subject to a specific set of tuning parameter values, and the validation sample is used for tuning the hyperparameters. The model to be estimated over the training sample is

$$y_{t+h} = G^e\left(\mathcal{X}_t, \boldsymbol{\beta}_{h,t}\right) + \epsilon_{t+h}.$$

where y_{t+h} is a time series indexed by j whose value in period $h \geq 1$ the machine is asked to predict at time t, $\mathcal{X}_t$ is a large input dataset of right-hand-side variables including the intercept, and $G^e(\cdot)$ is a machine learning estimator that can be represented by a (potentially) high-dimensional set of finite-valued parameters $\boldsymbol{\beta}^e_{h,t}$. We consider two estimators for $G^e(\cdot)$: Elastic Net $G^{\mathrm{EN}}(\mathcal{X}_t, \boldsymbol{\beta}^{\mathrm{EN}}_{j,h})$, and Long Short-Term Memory (LSTM) network $G^{\mathrm{LSTM}}(\mathcal{X}_t, \boldsymbol{\beta}^{\mathrm{LSTM}}_{j,h})$. The $e \in \{\mathrm{EN}, \mathrm{LSTM}\}$ superscripts on $\boldsymbol{\beta}$ indicate that the parameters depend on the estimator being used (See the next section for a description of EN and LSTM). $\mathcal{X}_t$ always denotes the most recent data that would have been in real time prior to the date on which the forecast was submitted. To ensure that the effect of each variable in the input vector is regularized fairly during the estimation, we standardize the elements of $\mathcal{X}_t$ such that sample means are zero and sample standard deviations are unity. It should be noted that the most recent observation on the left-hand-side is generally available in real time only with a one-period lag, thus the forecasting estimations can only be run with data over a sample that stops one period later than today in real time. The parameters $\boldsymbol{\beta}^e_{h,t}$ are estimated by minimizing the mean-square loss function over the training sample with L_1 and L_2 penalties

$$L(\boldsymbol{\beta}^e_{h,t}, \mathbf{X}_{T_E}, \boldsymbol{\lambda}^e_t) \equiv \underbrace{\frac{1}{T_E}\sum_{\tau=1}^{T_E}\left(y_{\tau+h} - G^e\left(\mathcal{X}_\tau, \boldsymbol{\beta}^e_{h,t}\right)\right)^2}_{\text{Mean Square Error}} + \underbrace{\lambda^e_{1,t}\sum_{k=1}^{K}\left|\boldsymbol{\beta}^e_{j,h,t,k}\right|}_{L_1 \text{ Penalty}} + \underbrace{\lambda^e_{2,t}\sum_{k=1}^{K}(\boldsymbol{\beta}^e_{j,h,t,k})^2}_{L_2 \text{ Penalty}}$$

where $\mathbf{X}_{T_E} = (y_{t-T_E}, \ldots, y_t, \mathcal{X}'_{t-T_E}, \ldots, \mathcal{X}'_t)'$ is the vector containing all observations in the training sample of size T_E. The estimated $\boldsymbol{\beta}^e_{h,t}$ is a function of the data $\boldsymbol{X}_{T_E}$ and a non-negative regularization parameter vector $\boldsymbol{\lambda}^e_t = \left(\lambda^e_{1,t}, \lambda^e_{2,t}, \boldsymbol{\lambda}^{LSTM}_{0,t}\right)'$ where $\boldsymbol{\lambda}^{LSTM}_{0,t}$ is a set of hyperparameters only relevant when using the LSTM estimator for $G^e(\cdot)$ (see below). For the EN case there are only two hyperparameters, which determine the optimal shrinkage and sparsity of the time t machine specification. The regularization parameters $\boldsymbol{\lambda}^e_t$ are estimated by minimizing the mean-square loss over pseudo-out-of-sample forecast errors generated from rolling regressions through the validation sample:

$$\widehat{\boldsymbol{\lambda}}^e_t, \widehat{T}_E, \widehat{T}_V = \underset{\boldsymbol{\lambda}^e_t, T_E, T_V}{argmin}\left\{\frac{1}{T_V - h}\sum_{\tau=T_E}^{T_E+T_V-h}\left(y_{\tau+h} - G^e(\mathcal{X}_\tau, \widehat{\boldsymbol{\beta}}^e_{j,h,\tau}(\mathbf{X}_{T_E}, \boldsymbol{\lambda}^e_t))\right)^2 + \underbrace{\lambda^e_{1,t}\sum_{k=1}^{K}\left|\boldsymbol{\beta}^e_{j,h,t,k}\right|}_{L_1 \text{ Penalty}} + \underbrace{\lambda^e_{2,t}\sum_{k=1}^{K}(\boldsymbol{\beta}^e_{j,h,t,k})^2}_{L_2 \text{ Penalty}}\right\}$$

where $\widehat{\boldsymbol{\beta}}^e_{j,h,\tau}(\cdot)$ for $e \in \{\text{EN}, \text{LSTM}\}$ is the time τ estimate of $\boldsymbol{\beta}^e_{j,h}$ given $\boldsymbol{\lambda}^e_t$ and data through time τ in a training sample of size T_E. Denote the combined final estimator $\widehat{\boldsymbol{\beta}}^e_{h,t}(\boldsymbol{X}_{\widehat{T}_E}, \widehat{\boldsymbol{\lambda}}^e_t)$, where the regularization parameter $\widehat{\boldsymbol{\lambda}}^e_t$ is estimated using cross-validation dynamically over time. Note that the algorithm also asks the machine to dynamically choose both the optimal training window $\widehat{T}_E$ and the optimal validation window $\widehat{T}_V$ by minimizing the pseudo-out-of-sample MSE.

The estimation of $\widehat{\boldsymbol{\beta}}^e_{h,t}(\boldsymbol{X}_{\widehat{T}_E}, \widehat{\boldsymbol{\lambda}}^e_t)$ is repeated sequentially in rolling subsamples, with parameters estimated from information known at time t. Note that the time t subscripts of $\widehat{\boldsymbol{\beta}}^e_{h,t}$ and $\widehat{\boldsymbol{\lambda}}^e_t$ denote one in a sequence of time-invariant parameter estimates obtained from rolling subsamples, rather than estimates that vary over time within a sample. Likewise, we denote the time t machine belief about y_{t+h} as $\mathbb{E}^e_t[y_{t+h}]$, defined by

$$\mathbb{E}^e_t[y_{t+h}] \equiv G^e\left(\mathcal{X}_t, \widehat{\boldsymbol{\beta}}^e_{h,t}(\boldsymbol{X}_{\widehat{T}_E}, \widehat{\boldsymbol{\lambda}}^e_t)\right)$$

Finally, the machine MSE is computed by averaging across the sequence of squared forecast errors in the true out-of-sample forecasts for periods $t = (\dot{T} + h), \ldots, T$ where T is the last period of our sample. The true out-of-sample forecasts used for neither estimation nor tuning is the *testing subsample* used to evaluate the model's predictive performance.

On rare occasions, one or more of the explanatory variables used in the machine forecast specification assumes a value that is order of magnitudes different from its historical value. This is usually indicative of a measurement problem in the raw data. We therefore program the machine to detect in real-time whether its forecast is an extreme outlier, and in that case to discard the forecast replacing it with the historical mean. Specifically, at each t, the machine forecast $\mathbb{E}^e_t[y_{t+h}]$ is set to be the historical mean calculated up to time t whenever the former is five or more standard deviations above its own rolling mean over the most recent 20 quarters.

We include the contemporaneous survey forecasts $\mathbb{F}_t[y_{t+h}]$ for the median respondent only for inflation and GDP forecasts, following Bianchi et al. (2022). This procedure allows the machine to capture intangible information due to judgement or private signals. Specifically, for these forecasts of inflation and GDP growth, we consider the following machine learning empirical specification for forecasting y_{t+h} given information at time t, to be benchmarked against the time t survey forecast of respondent-type X, where this type is the median here:

$$y_{t+h} = G^e_{jh}(\mathbf{Z}_t) + \gamma_{jh\mathbb{M}}\mathbb{F}_t[y_{t+h}] + \epsilon_{t+h}, \qquad h \geq 1$$

where $\gamma_{jh\mathbb{M}}$ is a parameter to be estimated, and where $G_{jh\mathbb{M}}(\mathbf{Z}_t)$ represents a ML estimator as function of big data. Note that the intercept α_{jh} from Bianchi et al. (2022) gets absorbed into the $G^e_{jh}(\mathbf{Z}_t)$ in LSTM via the outermost bias term.

OE.1.1 Elastic Net (EN)

We use the Elastic Net (EN) estimator, which combines Least Absolute Shrinkage and Selection Operator (LASSO) and ridge type penalties. The model can be written as:

$$y_{t+h} = \mathcal{X}'_{tj}\boldsymbol{\beta}^{\text{EN}}_{j,h} + \epsilon_{t+h}$$

where $\mathcal{X}_t = (1, \mathcal{X}_{1t,...,}\mathcal{X}_{Kt})'$ include the independent variable observations $(\mathbb{F}_t[y_{t+h}], \mathcal{Z}_{j,t})$ into a vector with "1" and $\boldsymbol{\beta}_{j,h}^{\text{EN}} = (\alpha_{j,h}, \beta_{j,h\mathbb{F}}, vec(\mathbf{B}_{j,h\mathcal{Z}}))' \equiv (\beta_0, \beta_1, ...\beta_K)'$ collects all the coefficients.

It is customary to standardize the elements of $\mathcal{X}_t$ such that sample means are zero and sample standard deviations are unity. The coefficient estimates are then put back in their original scale by multiplying the slope coefficients by their respective standard deviations, and adding back the mean (scaled by slope coefficient over standard deviation.) The EN estimator incorporates both an L_1 and L_2 penalty:

$$\widehat{\boldsymbol{\beta}}_{j,h}^{\text{EN}} = \underset{\beta_0,\beta_1,...,\beta_K}{argmin} \frac{1}{T_E}\sum_{\tau=1}^{T_E}\left(y_{\tau+h} - \mathcal{X}_\tau'\boldsymbol{\beta}_{j,h}\right)^2 + \underbrace{\lambda_1\sum_{k=1}^{K}|\boldsymbol{\beta}_{j,h,k}|}_{\text{LASSO}} + \underbrace{\lambda_2\sum_{k=1}^{K}(\boldsymbol{\beta}_{j,h,k})^2}_{\text{ridge}}$$

By minimizing the MSE over the training samples, we choose the optimal λ_1 and λ_2 values simultaneously.

In the implementation, the EN estimator is sometimes used as an input into the algorithm using the LSTM estimator. Specifically, we ensure that the machine forecast can only differ from the relevant benchmark if it demonstrably improves the pseudo out-of-sample prediction in the training samples *prior* to making a true out-of-sample forecast. Otherwise, the machine is replaced by the benchmark calculated up to time t. In some cases the benchmark is a survey forecast, in others it could be a historical mean value for the variable. However, for the implementation using LSTM, we also use the EN forecast as a benchmark.

OE.1.2 Long Short-Term Memory (LSTM) Network

An LSTM network is a type of Recurrent Neural Network (RNN), which are neural networks used to learn about sequential data such as time series or natural language. In particular, LSTM networks can learn long-term dependencies between across time periods by introducing hidden layers and memory cells to control the flow of information over longer time periods. The general case of the LSTM network with up to N hidden layers is defined as

$$\underbrace{G^{\text{LSTM}}(\mathcal{X}_t, \boldsymbol{\beta}_{j,h}^{\text{LSTM}})}_{1\times1} = \underbrace{W^{(yh^N)}}_{1\times D_{h^N}}\underbrace{h_t^N}_{D_{h^N}\times1} + \underbrace{b_y}_{1\times1} \quad \text{(Output layer)}$$

$$\underbrace{h_t^n}_{D_{h^n}\times1} = \underbrace{o_t^n}_{D_{h^n}\times1} \odot \tanh(\underbrace{c_t^n}_{D_{h^n}\times1}) \quad \text{(Hidden layer)}$$

$$\underbrace{c_t^n}_{D_{h^n}\times1} = \underbrace{f_t^n}_{D_{h^n}\times1} \odot \underbrace{c_{t-1}^n}_{D_{h^n}\times1} + \underbrace{i_t^n}_{D_{h^n}\times1} \odot \underbrace{\widetilde{c}_t^n}_{D_{h^n}\times1} \quad \text{(Final memory)}$$

$$\underbrace{\widetilde{c}_t^n}_{D_{h^n}\times1} = \tanh(\underbrace{W^{(c^nh^{n-1})}}_{D_{h^n}\times D_{h^{n-1}}}\underbrace{h_t^{n-1}}_{D_{h^{n-1}}\times1} + \underbrace{W^{(c^nh^n)}}_{D_{h^n}\times D_{h^n}}\underbrace{h_{t-1}^n}_{D_{h^n}\times1} + \underbrace{b_{c^n}}_{D_{h^n}\times1}) \quad \text{(New memory)}$$

$$\underbrace{f_t^n}_{D_{h^n}\times1} = \sigma(\underbrace{W^{(f^nh^{n-1})}}_{D_{h^n}\times D_{h^{n-1}}}\underbrace{h_t^{n-1}}_{D_{h^{n-1}}\times1} + \underbrace{W^{(f^nh^n)}}_{D_{h^n}\times D_{h^n}}\underbrace{h_{t-1}^n}_{D_{h^n}\times1} + \underbrace{b_{f^n}}_{D_{h^n}\times1}) \quad \text{(Forget gate)}$$

$$\underbrace{i_t^n}_{D_{h^n}\times1} = \sigma(\underbrace{W^{(i^nh^{n-1})}}_{D_{h^n}\times D_{h^{n-1}}}\underbrace{h_t^{n-1}}_{D_{h^{n-1}}\times1} + \underbrace{W^{(i^nh^n)}}_{D_{h^n}\times D_{h^n}}\underbrace{h_{t-1}^n}_{D_{h^n}\times1} + \underbrace{b_{i^n}}_{D_{h^n}\times1}) \quad \text{(Input gate)}$$

$$\underbrace{o_t^n}_{D_{h^n}\times1} = \sigma(\underbrace{W^{(o^nh^{n-1})}}_{D_{h^n}\times D_{h^{n-1}}}\underbrace{h_t^{n-1}}_{D_{h^{n-1}}\times1} + \underbrace{W^{(o^nh^n)}}_{D_{h^n}\times D_{h^n}}\underbrace{h_{t-1}^n}_{D_{h^n}\times1} + \underbrace{b_{o^n}}_{D_{h^n}\times1}) \quad \text{(Output gate)}$$

where $n = 1, \ldots, N$ indexes each hidden layer. $h_t^n \in \mathbb{R}^{D_{h^n}}$ is the n-th *hidden layer*, where D_{h^n} is the number of *neurons* or *nodes* in the hidden layer. The 0-th layer is defined as the input data: $h_t^0 \equiv \mathcal{X}_t$. The memory cell c_t^n allows the LSTM network to retain information over longer time periods. The output gate o_t^n controls the extent to which the memory cell c_t^n maps to the hidden layer h_t^n. The forget gate f_t^n controls the flow of information carried over from the final memory in the previous timestep c_{t-1}^n. The input gate i_t^n controls the flow of information from the new memory cell $\widetilde{c}_t^n$. The initial states for the hidden layers $(h_0^n)_{n=1}^N$ and memory cells $(c_0^n)_{n=1}^N$ are set to zeros. $\sigma(\cdot)$ and $\tanh(\cdot)$ are *activation functions* that introduce non-linearities in the LSTM network, applied elementwise. $\sigma : \mathbb{R} \to \mathbb{R}$ is the sigmoid function: $\sigma(x) = (1 + e^{-x})^{-1}$. $\tanh : \mathbb{R} \to \mathbb{R}$ is the hyperbolic tangent function: $\tanh(x) = \frac{e^{2x}-1}{e^{2x}+1}$. The $\odot$ operator refers to elementwise multiplication. $\boldsymbol{\beta}_{j,h}^{\text{LSTM}} \equiv (((vec(W^{(g^nh^{n-1})})', vec(W^{(g^nh^n)})', b_{g^n}')_{g\in\{c,f,i,o\}})_{n=1}^N, vec(W^{(yh^N)})', b_y)'$ are parameters to be estimated. We will refer to parameters indexed with W as *weights*; parameters indexed with b are *biases*. We estimate the parameters $\boldsymbol{\beta}_{j,h}^{\text{LSTM}}$ for the LSTM network using Stochastic Gradient Decent (SGD), which is an iterative algorithm for minimizing the loss function and proceeds as follows:

1. *Initialization.* Fix a random seed R and draw a starting value of the parameters $\boldsymbol{\beta}_{j,h}^{(0)}$ randomly, where the superscript (0) in parentheses indexes the iteration for an estimate of $\boldsymbol{\beta}_{j,h}^{\text{LSTM}}$.

(a) Initialize input weights $W^{(g^n h^{n-1})} \in \mathbb{R}^{D_{h^n} \times D_{h^{n-1}}}$ for $g \in \{c, f, i, o\}$ using the *Glorot* initializer. Draw from a uniform distribution with zero mean and a variance that depends on the dimensions of the matrix:

$$W_{ij}^{(g^n h^{n-1})} \overset{iid}{\sim} U\left[-\sqrt{\frac{6}{D_{h^n} + D_{h^{n-1}}}}, \sqrt{\frac{6}{D_{h^n} + D_{h^{n-1}}}}\right]$$

for each $i = 1, \ldots, D_{h^n}$ and $j = 1, \ldots, D_{h^{n-1}}$.

(b) Initialize the recurrent weights $W^{(g^n h^n)} \in \mathbb{R}^{D_{h^n} \times D_{h^n}}$ for $g \in \{c, f, i, o\}$ using the *Orthogonal* initializer. Use the orthogonal matrix obtained from the QR decomposition of a $D_{h^n} \times D_{h^n}$ matrix of random numbers drawn from a standard normal distribution.

(c) Initialize biases $(b_{g^n})_{g \in \{c,f,i,o\}}$, hidden layers h_0^n, and memory cells c_0^n with zeros.

2. *Mini-batches.* Prepare the input data by dividing the training sample into a collection of *mini-batches.*

(a) Suppose that we have a multi-variate time-series training sample with dimensions (T_E, K) whose time steps t are indexed by $t = 1, \ldots, T_E$ and K is the number of predictors. We transform this training sample into a 3-D tensor with dimensions (N_S, M, K) where

- N_S = Total number of sequences in training sample
- M = Sequence length, i.e., number of time steps in each sequence
- K = Input size, i.e., number of predictors in each time step

This can be done by creating overlapping sequences from the time series:

- Sequence 1 contains time steps $1, \ldots, M$
- Sequence 2 contains time steps $2, \ldots, M + 1$
- Sequence 3 contains time steps $3, \ldots, M + 2$
- ...
- Sequence $T_E - M$ contains time steps $T_E - M, \ldots, T_E - 1$
- Sequence $N_S = T_E - M + 1$ contains time steps $T_E - M + 1, \ldots, T_E$

(b) Randomly shuffle the N_S sequences by randomly sampling a permutation without replacement.

(c) Partition the N_S shuffled sequences into $\lceil N_S/N_B \rceil$ mini-batches. We partition the N_S sequences in the training sample ((N_S, M, K) tensor) into a list of $\lceil N_S/N_B \rceil$ mini-batches. A mini-batch is a (N_B, M, K)-dimensional tensor containing N_B out of N_S randomly shuffled sequences. When N_S/N_B is not a whole number, $\lfloor N_S/N_B \rfloor$ of the mini-batches will be 3-D tensors with dimensions (N_B, M, K). One batch will contain leftover sequences and will have dimensions $(N_S \% N_B, M, K)$ where % is the modulus operator. Let $B^{(1)}, \ldots, B^{\lceil N_S/N_B \rceil}$ denote the list of mini-batches.

- N_S = Total number of sequences in training sample
- N_B = Mini-batch size, i.e., number of sequences in each partition.
- M = Sequence length, i.e., number of time steps in each sequence
- K = Input size, i.e., number of predictors in each time step

3. Repeat until the stopping condition is satisfied $(k = 1, 2, 3, \ldots)$:

(a) *Dropout.* Apply dropout to the mini-batch. To obtain the n-th hidden layer under dropout, multiply the current value of the $n-1$-th hidden layer h_t^{n-1} and the lagged value of the n-th hidden layer h_{t-1}^n with binary masks $r_{t,h_t^{n-1}}^{(k)} \in \mathbb{R}^{D_{h^{n-1}}}$ and $r_{t,h_{t-1}^n}^{(k)} \in \mathbb{R}^{D_{h^n}}$, respectively:

$$\underbrace{\overline{h}_t^{n-1}}_{D_{h^{n-1}} \times 1} = \underbrace{r_{t,h_t^{n-1}}^{(k)}}_{D_{h^{n-1}} \times 1} \odot \underbrace{h_t^{n-1}}_{D_{h^{n-1}} \times 1}, \quad r_{t,h_t^{n-1},i}^{(k)} \overset{iid}{\sim} Bernoulli(p_{h_t^{n-1}}), \quad i = 1, \ldots, D_{h^{n-1}}$$

$$\underbrace{\overline{h}_{t-1}^{n}}_{D_{h^n} \times 1} = \underbrace{r_{t,h_{t-1}^n}^{(k)}}_{D_{h^n} \times 1} \odot \underbrace{h_{t-1}^n}_{D_{h^n} \times 1}, \quad r_{t,h_{t-1}^n,i}^{(k)} \overset{iid}{\sim} Bernoulli(p_{h_{t-1}^n}), \quad i = 1, \ldots, D_{h^n}$$

where $t \in B^{(k)}$ and $n = 1, \ldots, N$ indexes the hidden layer and it is understood that the 0-th layer is the input vector $h_t^0 \equiv \mathcal{X}_t$. $p_{h_t^{n-1}}, p_{h_{t-1}^n} \in [0, 1]$ is the probability that time t nodes in the $n-1$-th hidden layer and time $t-1$ nodes in the n-th hidden layer are retained, respectively.

(b) *Stochastic Gradient.* Average the gradient over observations in the mini-batch

$$\nabla L(\boldsymbol{\beta}_{j,h}^{(k-1)}, \mathbf{X}_{B^{(k)}}, \boldsymbol{\lambda}^{\text{LSTM}}) = \frac{1}{M} \sum_{t \in B^{(k)}} \nabla L(\boldsymbol{\beta}_{j,h}^{(k-1)}, \mathbf{X}_t, \boldsymbol{\lambda}^{\text{LSTM}})$$

where $\nabla L(\boldsymbol{\beta}_{j,h}^{(k-1)}, \mathbf{X}_t, \boldsymbol{\lambda}^{\text{LSTM}})$ is the gradient of the loss function with respect to the parameters $\boldsymbol{\beta}_{j,h}^{(k-1)}$, evaluated at the time t observation $\mathbf{X}_t = (y_{t+h}, \widehat{\mathcal{X}}_t')'$ after applying dropout.

(c) *Learning rate shrinkage.* Update the parameters to $\boldsymbol{\beta}_{j,h}^{(k)}$ using the Adaptive Moment Estimation (Adam) algorithm. The method uses the first and second moments of the gradients to shrink the overall learning rate to zero as the gradient approaches zero.

$$\boldsymbol{\beta}_{j,h}^{(k)} = \boldsymbol{\beta}_{j,h}^{(k-1)} - \gamma \frac{m^{(k)}}{\sqrt{v^{(k)}} + \varepsilon}$$

where $m^{(k)}$ and $v^{(k)}$ are weighted averages of first two moments of past gradients:

$$m^{(k)} = \frac{1}{1 - \pi_1^k} (\pi_1 m^{(k-1)} + (1 - \pi_1) \nabla L(\boldsymbol{\beta}_{j,h}^{(k-1)}, \mathbf{X}_{B^{(k)}}, \boldsymbol{\lambda}^{\text{LSTM}}))$$

$$v^{(k)} = \frac{1}{1 - \pi_2^k} (\pi_2 v^{(k-1)} + (1 - \pi_2) \nabla L(\boldsymbol{\beta}_{j,h}^{(k-1)}, \mathbf{X}_{B^{(k)}}, \boldsymbol{\lambda}^{\text{LSTM}})^2)$$

π^k denotes the k-the power of $\pi \in (0, 1)$, and $/$, $\sqrt{\cdot}$, and $(\cdot)^2$ are applied elementwise. The default values of the hyperparameters are $m^{(0)} = v^{(0)} = 0$ (initial moment vectors), $\gamma = 0.001$ (initial learning rate), $(\pi_1, \pi_2) = (0.9, 0.999)$ (decay rates), and $\varepsilon = 10^{-7}$ (prevent zero denominators).

(d) *Stopping Criteria.* Stop iterating and return $\boldsymbol{\beta}_{j,h}^{(k)}$ if one of the following holds:

- *Early stopping.* At each iteration, use the updated $\boldsymbol{\beta}_{j,h}^{(k)}$ to calculate the loss from the validation sample. Stop when the validation loss has not improved for S steps, where S is a "patience" hyperparameter. By updating the parameters for fewer iterations, early stopping shrinks the final parameters $\boldsymbol{\beta}_{j,h}$ towards the initial guess $\boldsymbol{\beta}_{j,h}^{(0)}$, and at a lower computational cost than ℓ_2 regularization.
- *Maximum number of epochs.* Stop if the number of iterations reaches the maximum number of epochs E. An epoch happens when the full set of the training sample has been used to update the parameters. If the training sample has T_E observations and each mini-batch has M observations, then each epoch would contain $\lceil T_E/M \rceil$ iterations (after rounding up as needed). So the maximum number of iterations is bounded by $E \times \lceil T_E/M \rceil$.

4. *Ensemble forecasts.* Repeat steps 1. and 2. over different random seeds R and save each of the estimated parameters $\widehat{\boldsymbol{\beta}}_{j,h,T_E}^{LSTM}(\boldsymbol{X}_{T_E}, \boldsymbol{\lambda}^{\text{LSTM}}, R)$. Then construct out-of-sample forecasts using the top 10 out of 20 starting values with the best performance in the validation sample. Ensemble can be considered as a regularization method because it aims to guard against overfitting by shrinking the forecasts toward the average across different random seeds. The random seed affects the random draws of the parameter's initial starting value $\boldsymbol{\beta}_{j,h}^{(0)}$, the sequences selected in each mini-batch $B^{(k)}$, and the dropout mask $r_t^{(k)}$.

Hyperparameters Let $\boldsymbol{\lambda}^{\text{LSTM}} \equiv [\lambda_1, \lambda_2, \gamma, \pi_1, \pi_2, p, N, (D_{h^n})_{n=1}^N, M, E, S]'$ collect all the hyper-parameters that control the LSTM network's complexity and prevent the model from overfitting the data. The number of hidden layers N and the number of neurons $D_{h^1}, \ldots, D_{h^N}$ in each hidden layer are hyper-parameters that characterize the network's architecture. To choose the number of neurons in each layer, we apply a geometric pyramid rule where the dimension of each additional hidden layer is half that of the previous hidden layer. We select the best LSTM architecture iteratively by minimizing the pseudo out-of-sample mean-squared error from rolling forecasts over the validation sample. Table OA.12 reports the hyper-parameters for the LSTM network and its estimation. Hyper-parameters reported as a range or a set of values are cross-validated. The hyper-parameters are estimated by minimizing the mean-square loss over pseudo out-of-sample forecast errors generated from rolling regressions through the validation sample. The pseudo out-of-sample forecasts are ensemble averages implied by parameters based on different random seeds R.

Adaptive Architecture Selection We allow the LSTM architecture to evolve over time using a simple, adaptive updating procedure. At each period in the testing sample, the machine selects the architecture (number of hidden layers and neurons per layer) that minimized out-of-sample forecast errors in the preceding period. The candidate architectures considered span various combinations of hidden layers and neurons per layer, as listed in Table OA.12. The architecture is updated quarterly by using the forecast performance from the most recent quarter. This approach allows the machine to adjust its specification over time based on evolving patterns in the data, while avoiding look-ahead bias or overfitting to future outcomes.

Table OA.12: Candidate hyper-parameters for the machine learning forecast

Variable	Earnings Growth	Stock Returns
Horizon (Years)	1,2,3,4,5	1,2,3,4,5
(a) Elastic Net		
L_1 penalty λ_1	$[10^{-2}, 10^{1}]$	$[10^{-6}, 10^{-2}]$
L_2 penalty λ_2	$[10^{-2}, 10^{1}]$	$[10^{-6}, 10^{-2}]$
Training window T_E	$4, 6, 8, 10$	$5, 7$
Validation window T_V	$4, 6, 8, 10$	$5, 7, 20$
(b) Long Short-Term Memory Network		
L_1 penalty λ_1	$[10^{-6}, 10^{-2}]$	$[10^{-6}, 10^{-2}]$
L_2 penalty λ_2	$[10^{-6}, 10^{-2}]$	$[10^{-6}, 10^{-2}]$
Learning rate γ	0.001	0.001
Gradient decay π_1, π_2	$0.9, 0.999$	$0.9, 0.999$
Dropout input p_x	0.5	0.5
Dropout recurrent p_h	0.5	0.5
Hidden layers N	$1, 3, 5$	$1, 3, 5$
Neurons per layer	$16, 32, 64$	$4, 8, 16$
Mini-batch size M	4	4
Max epochs E	$10,000$	$10,000$
Early stopping S	20	20
Random seeds R	$1, \ldots, 20$	$1, \ldots, 20$
Training window T_E	$4, 8, 12$	$5, 7$
Validation window T_V	$4, 8, 12$	$5, 7, 20$

Notes: This table reports the hyperparameters considered in the machine learning algorithm for each estimator.

OE.2 Data Inputs for Machine Learning Algorithm

OE.2.1 Macro Data Surprises

These data are used as inputs into the machine learning forecasts. I obtain median forecasts for GDP growth (Q/Q percentage change), core CPI (Month/Month change), unemployment rate (percentage point), and nonfarm payroll (month/month change) from the Money Market Service Survey. The median market survey forecasts are compiled and published by the Money Market Services (MMS) the Friday before each release. I apply the approach used in Bauer and Swanson (2023) and define macroeconomic data surprise as the actual value of the data release minus the median expectation from MMS on the Friday immediately prior to that data release. The GDP growth forecasts are available quarterly from 1990Q1 to 2023Q4. The core CPI forecast is available monthly from July 1989 to December 2023. The median forecasts for the unemployment rate and nonfarm payrolls are available monthly from January 1980 to December 2023, and January 1985 to December 2023, respectively. All survey forecasts were downloaded from Haver Analytics on December 17, 2022 and the Bloomberg Terminal on July 15, 2025. To pin down the timing of when the news was actually released I follow the published tables of releases from the Bureau of Labor Statistics (BLS), discussed below.

The macro news events are indexed by their date and time of the data release, while the machine learning algorithm is adapted to quarterly sampling frequencies. When including the macro data surprises as additional predictors for the machine forecast, I time-aggregate the macro data surprises to a quarterly frequency by taking the sum of the surprises across data releases that occurred before the response deadline set for the machine. For example, if the response deadline is set to the first day of the middle month of each quarter (e.g., February 1st), I take the sum of the surprises from data releases up to the day before the deadline, the last day of the first month of each quarter (e.g., January 31st).

OE.2.2 FOMC Surprises

FOMC surprises are defined as the changes in the current-month, 1, 2, 6, 12, and 24 month-ahead federal funds futures (FFF) contract rate and changes in the 1, 2, 4, and 8 quarter-ahead Eurodollar (ED) futures contract rate, from 10 minutes before to 20 minutes after each U.S. Federal Reserve Federal Open Market Committee (FOMC) announcement. The data on FFF and ED were downloaded on July 15, 2025. When benchmarking against a survey, I use the last FOMC meeting before the survey deadline to compute surprises. For surveys that do not have a clear deadline, I compute surprises using from the last FOMC in the first month of the quarter. When benchmarking against moving average, I use the last FOMC meeting before the end of the first month in each quarter to compute surprises.

When including the FOMC surprises as additional predictors for the machine forecast, I time-aggregate the FOMC surprises to a quarterly frequency by taking the sum of the surprises across FOMC announcements that occurred before the response deadline set for the machine. For example, if the response deadline is set to the first day of the middle month of each quarter (e.g., February 1st), I take the sum of the surprises from FOMC announcements up to the day before the deadline, the last day of the first month of each quarter (e.g., January 31st).

OE.2.3 S&P 500 Jumps

As a measure of the market's reaction to news shocks, I use the jump in the S&P 500 pre- and post- a 30-minute window around major news events. The events in our analysis include (i) 1,482 macroeconomic data releases for U.S. GDP, Consumer Price Index (CPI), unemployment, and payroll data spanning 1980:01-2023:12, (ii) 16 corporate earnings announcement days spanning 1999:03-2020:05, and (iii) 219 Federal Open Market Committee (FOMC) press releases from the Fed spanning 1994:02-2023:12. The corporate earnings news events are from Baker et al. (2019) who conduct textual analyses of *Wall Street Journal* articles to identify days in which there were large jumps in the aggregate stock market that could be attributed to corporate earnings news with high confidence. The jump in the S&P 500 for a given event is defined as $j_\tau = p_{\tau+\delta_{post}} - p_{\tau-\delta_{pre}}$, where τ indexes the time of an event and $p_\tau = \log(P_\tau)$ is the log S&P 500 index. δ_{pre} and δ_{post} denote the pre and post event windows, which is 10 minutes before and 20 minutes after the event, respectively. I obtain data on P_τ using tick-by-tick data on the S&P 500 index from tickdata.com. The series was purchased and downloaded on July 15, 2025 from `https://www.tickdata.com/`. I create the minutely data using the close price within each minute. I supplement the S&P 500 index using S&P500 E-mini futures for events that occur in off-market hours. I use the current-quarter contract futures. I purchased the S&P 500 E-mini futures from CME group on July 15, 2025 at `https://datamine.cmegroup.com/`. Our sample spans 1/2/1986 to 12/31/2023.

For each event, I separate out the events for which the S&P 500 increased over the window ($j_\tau^{(+)} \geq 0$) and those for which the market decreased ($j_\tau^{(-)} \leq 0$). I aggregate the event-level jumps to monthly time series by summing over all the relevant events within the month, where the events are partitioned into two groups based on the sign of the jump: $J_t^{(+)} = \sum_{\tau \in x(t)} j_\tau^{(+)}$, $J_t^{(-)} = \sum_{\tau \in x(t)} j_\tau^{(-)}$, where t indexes the month and $x(t)$ is the set of all events that occurred within month t. The procedure results in two monthly variables, $J_t^{(+)}$ and $J_t^{(-)}$, which capture total market reaction to news events in either direction during the quarter. The series spans the period 1994:02 to 2023:12. Separating out the events based on the sign of the jump allows us to capture any differential effects on return predictability based on whether the market perceived the news as good or bad. The partition also allows us to accurately capture the total extent of over- or underreaction. Otherwise, mixing all the events would only capture the net effect of the jumps and bias the market reaction towards zero.

When used as additional predictors in the for the machine forecast, the jumps need to be converted to quarterly time series because the machine learning algorithm is adapted to a quarterly sampling frequency. The set of events in $x(t)$ is chosen so that the machine only sees the news events that would have been available to the real-time firm. When combining the events within a quarter, I impose the response deadline used to produce the machine forecast. For example, if the response deadline is set to the first day of the middle month of each quarter (e.g., February 1st), I use the jumps from the events up to the day before the deadline, the last day of the first month of each quarter (e.g., January 31st).

OE.2.4 Real-Time Macro Data

This section gives details on the real time macro data inputs used in the machine learning forecasts. A subset of these series are used in the structural estimation. At each forecast date in the sample, I construct a dataset of macro variables that could have been observed on or before the day of the survey deadline. I use the Philadelphia Fed's Real-Time Data Set to obtain vintages of macro variables. The real-time data sets are available at https://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/data-files. These vintages capture changes to historical data due to periodic revisions made by government statistical agencies. The vintages for a particular series can be available at the monthly and/or quarterly frequencies, and the series have monthly and/or quarterly observations. In cases where a variable has both frequencies available for its vintages and/or its observations, I choose one format of the variable. For instance, nominal personal consumption expenditures on goods is quarterly data with both monthly and quarterly vintages available; in this case, I use the version with monthly vintages.

Table OA.13 gives the complete list of real-time macro variables. Included in the table is the first available vintages for each variable that has multiple vintages. I do not include the last vintage because most variables have vintages through the present. For variables BASEBASAQVMD, NBRBASAQVMD, NBRECBASAQVMD, and TRBASAQVMD, the last available vintage is 2013Q2. Table OA.13 also lists the transformation applied to each variable to make them stationary before generating factors. Let $X_{i,t}$ denote variable i at time t after the transformation, and let $X_{i,t}^A$ be the untransformed series. Let $\Delta = (1 - L)$ with $LX_{i,t} = X_{it-1}$. There are seven possible transformations with the following codes:

1 Code lv: $X_{i,t} = X_{i,t}^A$

2 Code Δlv: $X_{i,t} = X_{i,t}^A - X_{it-1}^A$

3 Code $\Delta^2 lv$: $X_{i,t} = \Delta^2 X_{i,t}^A$

4 Code ln: $X_{i,t} = \ln(X_{i,t}^A)$

5 Code Δln: $X_{i,t} = \ln(X_{i,t}^A) - \ln(X_{it-1}^A)$

6 Code $\Delta^2 ln$: $X_{i,t} = \Delta^2 \ln(X_{i,t}^A)$

7 Code $\Delta lv/lv$: $X_{i,t} = (X_{i,t}^A - X_{it-1}^A)/X_{it-1}^A$

Table OA.13: List of Macro Dataset Variables

No.	Short Name	Source	Tran	Description	First Vintage
			Group 1: Output and Income		
1	IPMMVMD	Philly Fed	Δln	Ind. production index - Manufacturing	1962M11
2	IPTMVMD	Philly Fed	Δln	Ind. production index - Total	1962M11
3	CUMMVMD	Philly Fed	lv	Capacity utilization - Manufacturing	1979M8
4	CUTMVMD	Philly Fed	lv	Capacity utilization - Total	1983M7
5	NCPROFATMVQD	Philly Fed	Δln	Nom. corp. profits after tax without IVA/CCAdj	1965Q4
6	NCPROFATWMVQD	Philly Fed	Δln	Nom. corp. profits after tax with IVA/CCAdj	1981Q1
7	OPHMVQD	Philly Fed	Δln	Output per hour - Business sector	1998Q4
8	NDPIQVQD	Philly Fed	Δln	Nom. disposable personal income	1965Q4
9	NOUTPUTQVQD	Philly Fed	Δln	Nom. GNP/GDP	1965Q4
10	NPIQVQD	Philly Fed	Δln	Nom. personal income	1965Q4
11	NPSAVQVQD	Philly Fed	Δlv	Nom. personal saving	1965Q4
12	OLIQVQD	Philly Fed	Δln	Other labor income	1965Q4
13	PINTIQVQD	Philly Fed	Δln	Personal interest income	1965Q4
14	PINTPAIDQVQD	Philly Fed	Δln	Interest paid by consumers	1965Q4
15	PROPIQVQD	Philly Fed	Δln	Proprietors' income	1965Q4
16	PTAXQVQD	Philly Fed	Δln	Personal tax and nontax payments	1965Q4
17	RATESAVQVQD	Philly Fed	Δlv	Personal saving rate	1965Q4
18	RENTIQVQD	Philly Fed	Δlv	Rental income of persons	1965Q4
19	ROUTPUTQVQD	Philly Fed	Δln	Real GNP/GDP	1965Q4
20	SSCONTRIBQVQD	Philly Fed	Δln	Personal contributions for social insurance	1965Q4
21	TRANPFQVQD	Philly Fed	Δln	Personal transfer payments to foreigners	1965Q4
22	TRANRQVQD	Philly Fed	Δln	Transfer payments	1965Q4
23	CUUR0000SA0E	BLS	$\Delta^2 ln$	Energy in U.S. city avg., all urban consumers, not seasonally adj	
			Group 2: Employment		
24	EMPLOYMVMD	Philly Fed	Δln	Nonfarm payroll	1946M12
25	HMVMD	Philly Fed	lv	Aggregate weekly hours - Total	1971M9
26	HGMVMD	Philly Fed	lv	Agg. weekly hours - Goods-producing	1971M9
27	HSMVMD	Philly Fed	lv	Agg. weekly hours - Service-producing	1971M9
28	LFCMVMD	Philly Fed	Δln	Civilian labor force	1998M11
29	LFPARTMVMD	Philly Fed	lv	Civilian participation rate	1998M11
30	POPMVMD	Philly Fed	Δln	Civilian noninstitutional population	1998M11
31	ULCMVQD	Philly Fed	Δln	Unit labor costs - Business sector	1998Q4
32	RUCQVMD	Philly Fed	Δlv	Unemployment rate	1965Q4
33	WSDQVQD	Philly Fed	Δln	Wage and salary disbursements	1965Q4
			Group 3: Orders, Investment, Housing		
34	HSTARTSMVMD	Philly Fed	Δln	Housing starts	1968M2
35	RINVBFMVQD	Philly Fed	Δln	Real gross private domestic inv. - Nonresidential	1965Q4
36	RINVCHIMVQD	Philly Fed	Δlv	Real gross private domestic inv. - Change in private inventories	1965Q4
37	RINVRESIDMVQD	Philly Fed	Δln	Real gross private domestic inv. - Residential	1965Q4
38	CASESHILLER	S&P	Δln	Case-Shiller US National Home Price index/CPI	1987M1
			Group 4: Consumption		
39	NCONGMMVMD	Philly Fed	Δln	Nom. personal cons. exp. - Goods	2009M8
40	NCONHHMMVMD	Philly Fed	Δln	Nom. hh. cons. exp.	2009M8
41	NCONSHHMMVMD	Philly Fed	Δln	Nom. hh. cons. exp. - Services	2009M8
42	NCONSNPMMVMD	Philly Fed	Δln	Nom. final cons. exp. of NPISH	2009M8
43	RCONDMMVMD	Philly Fed	Δln	Real personal cons. exp. - Durables	1998M11
44	RCONGMMVMD	Philly Fed	Δln	Real personal cons. exp. - Goods	2009M8
45	RCONHHMMVMD	Philly Fed	Δln	Real hh. cons. exp.	2009M8
46	RCONMMVMD	Philly Fed	Δln	Real personal cons. exp. - Total	1998M11
47	RCONNDMVMD	Philly Fed	Δln	Real personal cons. exp. - Nondurables	1998M11
48	RCONSHHMMVMD	Philly Fed	Δln	Real hh. cons. exp. - Services	2009M8
49	RCONSMMVMD	Philly Fed	Δln	Real personal cons. exp. - Services	1998M11
50	RCONSNPMMVMD	Philly Fed	Δln	Real final cons. exp. of NPISH	2009M8
51	NCONGMVQD	Philly Fed	Δln	Nom. personal cons. exp. - Goods	2009Q3
52	NCONHHMVQD	Philly Fed	Δln	Nom. hh. cons. exp.	2009Q3
53	NCONSHHMVQD	Philly Fed	Δln	Nom. hh. cons. exp. - Services	2009Q3
54	NCONSNPMVQD	Philly Fed	Δln	Nom. final cons. exp. of NPISH	2009Q3
55	RCONDMVQD	Philly Fed	Δln	Real personal cons. exp. - Durable goods	1965Q4
56	RCONGMVQD	Philly Fed	Δln	Real personal cons. exp. - Goods	2009Q3
57	RCONHHMVQD	Philly Fed	Δln	Real hh. cons. exp.	2009Q3
58	RCONMVQD	Philly Fed	Δln	Real personal cons. exp. - Total	1965Q4
59	RCONNDMVQD	Philly Fed	Δln	Real personal cons. exp. - Nondurable goods	1965Q4
60	RCONSHHMVQD	Philly Fed	Δln	Real hh. cons. exp. - Services	2009Q3
61	RCONSMVQD	Philly Fed	Δln	Real personal cons. exp. - Services	1965Q4
62	RCONSNPMVQD	Philly Fed	Δln	Real final cons. exp. of NPISH	2009Q3
63	NCONQVQD	Philly Fed	Δln	Nom. personal cons. exp.	1965Q4
			Group 5: Prices		
64	PCONGMMVMD	Philly Fed	$\Delta^2 ln$	Price index for personal cons. exp. - Goods	2009M8

No.	Short Name	Source	Tran	Description	First Vintage
65	PCONHHMMVMD	Philly Fed	$\Delta^2 ln$	Price index for hh. cons. exp.	2009M8
66	PCONSHHMMVMD	Philly Fed	$\Delta^2 ln$	Price index for hh. cons. exp. - Services	2009M8
67	PCONSNPMMVMD	Philly Fed	$\Delta^2 ln$	Price index for final cons. exp. of NPISH	2009M8
68	PCPIMVMD	Philly Fed	$\Delta^2 ln$	Consumer price index	1998M11
69	PCPIXMVMD	Philly Fed	$\Delta^2 ln$	Core consumer price index	1998M11
70	PPPIMVMD	Philly Fed	$\Delta^2 ln$	Producer price index	1998M11
71	PPPIXMVMD	Philly Fed	$\Delta^2 ln$	Core producer price index	1998M11
72	PCONGMVQD	Philly Fed	$\Delta^2 ln$	Price index for personal cons. exp. - Goods	2009Q3
73	PCONHHMVQD	Philly Fed	$\Delta^2 ln$	Price index for hh. cons. exp.	2009Q3
74	PCONSHHMVQD	Philly Fed	$\Delta^2 ln$	Price index for hh. cons. exp. - Services	2009Q3
75	PCONSNPMVQD	Philly Fed	$\Delta^2 ln$	Price index for final cons. exp. of NPISH	2009Q3
76	PCONXMVQD	Philly Fed	$\Delta^2 ln$	Core price index for personal cons. exp.	1996Q1
77	CPIQVMD	Philly Fed	$\Delta^2 ln$	Consumer price index	1994Q3
78	PQVQD	Philly Fed	$\Delta^2 ln$	Price index for GNP/GDP	1965Q4
79	PCONQVQD	Philly Fed	$\Delta^2 ln$	Price index for personal cons. exp.	1965Q4
80	PIMPQVQD	Philly Fed	$\Delta^2 ln$	Price index for imports of goods and services	1965Q4
		Group 6: Trade and Government			
81	REXMVQD	Philly Fed	Δln	Real exports of goods and services	1965Q4
82	RGMVQD	Philly Fed	Δln	Real government cons. and gross inv. - Total	1965Q4
83	RGFMVQD	Philly Fed	Δln	Real government cons. and gross inv. - Federal	1965Q4
84	RGSLMVQD	Philly Fed	Δln	Real government cons. and gross. inv. - State and local	1965Q4
85	RIMPMVQD	Philly Fed	Δln	Real imports of goods and services	1965Q4
86	RNXMVQD	Philly Fed	Δlv	Real net exports of goods and services	1965Q4
		Group 7: Money and Credit			
87	BASEBASAQVMD	Philly Fed	$\Delta^2 ln$	Monetary base	1980Q2
88	M1QVMD	Philly Fed	$\Delta^2 ln$	M1 money stock	1965Q4
89	M2QVMD	Philly Fed	$\Delta^2 ln$	M2 money stock	1971Q2
90	NBRBASAQVMD	Philly Fed	$\Delta lv/lv$	Nonborrowed reserves	1967Q3
91	NBRECBASAQVMD	Philly Fed	$\Delta lv/lv$	Nonborrowed reserves plus extended credit	1984Q2
92	TRBASAQVMD	Philly Fed	$\Delta^2 ln$	Total reserves	1967Q3
93	DIVQVQD	Philly Fed	Δln	Dividends	1965Q4

OE.2.5 Monthly Financial Data

The 147 financial series in this data set are versions of the financial dataset used in Jurado et al. (2015) and Ludvigson et al. (2021). It consists of a number of indicators measuring the behavior of a broad cross-section of asset returns, as well as some aggregate financial indicators not included in the macro dataset. These data include valuation ratios such as the dividend-price ratio and earnings-price ratio, growth rates of aggregate dividends and prices, default and term spreads, yields on corporate bonds of different ratings grades, yields on Treasuries and yield spreads, and a broad cross-section of industry equity returns. Following Fama and French (1992), returns on 100 portfolios of equities sorted into 10 size and 10 book-to-market categories. The dataset X^f also includes a group of variables we call "risk-factors," since they have been used in cross-sectional or time-series studies to uncover variation in the market risk-premium. These risk-factors include the three Fama and French (1993) risk factors, namely the excess return on the market MKT_t, the "small-minus-big" (SMB_t) and "high-minus-low" (HML_t) portfolio returns, the momentum factor UMD_t, and the small stock value spread $R15 - R11$.

The raw data used to form factors are always transformed to achieve stationarity. In addition, when forming forecasting factors from the large macro and financial datasets, the raw data (which are in different units) are standardized before performing PCA. When forming common uncertainty from estimates of individual uncertainty, the raw data (which are in this case in the same units) are demeaned, but we do not divide by the observation's standard deviation before performing PCA. Throughout, the factors are estimated by the method of static principal components (PCA). Specifically, the $T \times r_F$ matrix $\widehat{F}_t$ is $\sqrt{T}$ times the r_F eigenvectors corresponding to the r_F largest eigenvalues of the $T \times T$ matrix $xx'/(TN)$ in decreasing order. In large samples (when $\sqrt{T}/N \to \infty$), Bai and Ng (2006) show that the estimates $\widehat{F}_t$ can be treated as though they were observed in the subsequent forecasting regression. All returns and spreads are expressed in logs (i.e., the log of the gross return or spread), are displayed in percent (i.e., multiplied by 100), and are annualized by multiplying by 12. That is, if x is the original return or spread, we transform to $1200 \times \log(1+x/100)$. Federal Reserve data are annualized by default and are therefore not re-annualized. Note that this annualization implies that the annualized standard deviation (volatility) is equal to the data standard deviation divided by $\sqrt{12}$. The data series used in this dataset are listed below by data source. Additional details on data transformations are given below the table.

We convert monthly data to quarterly by using either the beginning-of-quarter or end-of-quarter values. The decision to use beginning-of-quarter or end-of-quarter depends on the survey deadline of a particular forecast date. If the survey deadline is known to be in the middle of the second month of quarter t, then it is conceivable that the forecasters would have information about the first month of quarter t. Therefore, we use the first month of that quarter's values. Alternatively, a few anomalous observations have unknown survey deadlines (e.g., the SPF deadlines for 1990Q1). In such cases, we allow only information up to quarter $t-1$ to enter the model. Thus, we use the last month of the previous quarter's values in these cases. Let $X_{i,t}$ denote variable i observed at time t after, e.g., logarithm and differencing transformation, and let $X_{i,t}^A$ be the actual (untransformed) series. Let $\Delta = (1-L)$ with $LX_{i,t} = X_{i,t-1}$. There are six possible transformations with the following codes:

$$1 \text{ Code } lv : X_{i,t} = X_{i,t}^A$$
$$2 \text{ Code } \Delta lv : X_{i,t} = X_{i,t}^A - X_{i,t-1}^A$$
$$3 \text{ Code } \Delta^2 lv : X_{i,t} = \Delta^2 X_{i,t}^A$$
$$4 \text{ Code } ln : X_{i,t} = \log(X_{i,t}^A)$$
$$5 \text{ Code } \Delta ln : X_{i,t} = \log(X_{i,t}^A) - \log(X_{i,t-1}^A)$$
$$6 \text{ Code } \Delta^2 ln : X_{i,t} = \Delta^2 \log(X_{i,t}^A)$$
$$7 \text{ Code } \Delta lv/lv : X_{i,t} = \frac{X_{i,t}^A - X_{i,t-1}^A}{X_{i,t-1}^A}$$

Table OA.14: List of Financial Dataset Variables

No.	Short Name	Source	Tran	Description
		Group 1: Prices, Yields, Dividends		
1	D_log(DIV)	CRSP	Δln	1 log D_t, see additional details below
2	D_log(P)	CRSP	Δln	1 log P_t, see additional details below
3	D_DIVreinvest	CRSP	Δln	1 log $D_t^{re,*}$, see additional details below
4	D_Preinvest	CRSP	Δln	1 log $P_t^{re,*}$, see additional details below
5	d-p	CRSP	ln	log $D_t - P_t$, see additional details below
		Group 2: Equity Risk Factors		
6	R15-R11	Kenneth French	lv	(Small, High) minus (Small, Low) sorted on (size, book-to-market)
7	Mkt-RF	Kenneth French	lv	Market excess return
8	SMB	Kenneth French	lv	Small Minus Big, sorted on size
9	HML	Kenneth French	lv	High Minus Low, sorted on book-to-market
10	UMD	Kenneth French	lv	Up Minus Down, sorted on momentum
		Group 3: Industries		
11	Agric	Kenneth French	lv	Agric industry portfolio
12	Food	Kenneth French	lv	Food industry portfolio
13	Beer	Kenneth French	lv	Beer industry portfolio
14	Smoke	Kenneth French	lv	Smoke industry portfolio
15	Toys	Kenneth French	lv	Toys industry portfolio
16	Fun	Kenneth French	lv	Fun industry portfolio
17	Books	Kenneth French	lv	Books industry portfolio
18	Hshld	Kenneth French	lv	Hshld industry portfolio
19	Clths	Kenneth French	lv	Clths industry portfolio
20	MedEq	Kenneth French	lv	MedEq industry portfolio
21	Drugs	Kenneth French	lv	Drugs industry portfolio
22	Chems	Kenneth French	lv	Chems industry portfolio
23	Rubbr	Kenneth French	lv	Rubbr industry portfolio
24	Txtls	Kenneth French	lv	Txtls industry portfolio
25	BldMt	Kenneth French	lv	BldMt industry portfolio
26	Cnstr	Kenneth French	lv	Cnstr industry portfolio
27	Steel	Kenneth French	lv	Steel industry portfolio
28	Mach	Kenneth French	lv	Mach industry portfolio
29	ElcEq	Kenneth French	lv	ElcEq industry portfolio
30	Autos	Kenneth French	lv	Autos industry portfolio
31	Aero	Kenneth French	lv	Aero industry portfolio
32	Ships	Kenneth French	lv	Ships industry portfolio
33	Mines	Kenneth French	lv	Mines industry portfolio
34	Coal	Kenneth French	lv	Coal industry portfolio
35	Oil	Kenneth French	lv	Oil industry portfolio
36	Util	Kenneth French	lv	Util industry portfolio
37	Telcm	Kenneth French	lv	Telcm industry portfolio
38	PerSv	Kenneth French	lv	PerSv industry portfolio
39	BusSv	Kenneth French	lv	BusSv industry portfolio
40	Hardw	Kenneth French	lv	Hardw industry portfolio
41	Chips	Kenneth French	lv	Chips industry portfolio
42	LabEq	Kenneth French	lv	LabEq industry portfolio
43	Paper	Kenneth French	lv	Paper industry portfolio
44	Boxes	Kenneth French	lv	Boxes industry portfolio
45	Trans	Kenneth French	lv	Trans industry portfolio
46	Whlsl	Kenneth French	lv	Whlsl industry portfolio

No.	Short Name	Source	Tran	Description
47	Rtail	Kenneth French	*lv*	Rtail industry portfolio
48	Meals	Kenneth French	*lv*	Meals industry portfolio
49	Banks	Kenneth French	*lv*	Banks industry portfolio
50	Insur	Kenneth French	*lv*	Insur industry portfolio
51	RlEst	Kenneth French	*lv*	RlEst industry portfolio
52	Fin	Kenneth French	*lv*	Fin industry portfolio
53	Other	Kenneth French	*lv*	Other industry portfolio
			Group 4: Size/BM	
54	1_2	Kenneth French	*lv*	(1, 2) portfolio sorted on (size, book-to-market)
55	1_4	Kenneth French	*lv*	(1, 4) portfolio sorted on (size, book-to-market)
56	1_5	Kenneth French	*lv*	(1, 5) portfolio sorted on (size, book-to-market)
57	1_6	Kenneth French	*lv*	(1, 6) portfolio sorted on (size, book-to-market)
58	1_7	Kenneth French	*lv*	(1, 7) portfolio sorted on (size, book-to-market)
59	1_8	Kenneth French	*lv*	(1, 8) portfolio sorted on (size, book-to-market)
60	1_9	Kenneth French	*lv*	(1, 9) portfolio sorted on (size, book-to-market)
61	1_high	Kenneth French	*lv*	(1, high) portfolio sorted on (size, book-to-market)
62	2_low	Kenneth French	*lv*	(2, low) portfolio sorted on (size, book-to-market)
63	2_2	Kenneth French	*lv*	(2, 2) portfolio sorted on (size, book-to-market)
64	2_3	Kenneth French	*lv*	(2, 3) portfolio sorted on (size, book-to-market)
65	2_4	Kenneth French	*lv*	(2, 4) portfolio sorted on (size, book-to-market)
66	2_5	Kenneth French	*lv*	(2, 5) portfolio sorted on (size, book-to-market)
67	2_6	Kenneth French	*lv*	(2, 6) portfolio sorted on (size, book-to-market)
68	2_7	Kenneth French	*lv*	(2, 7) portfolio sorted on (size, book-to-market)
69	2_8	Kenneth French	*lv*	(2, 8) portfolio sorted on (size, book-to-market)
70	2_9	Kenneth French	*lv*	(2, 9) portfolio sorted on (size, book-to-market)
71	2_high	Kenneth French	*lv*	(2, high) portfolio sorted on (size, book-to-market)
72	3_low	Kenneth French	*lv*	(3, low) portfolio sorted on (size, book-to-market)
73	3_2	Kenneth French	*lv*	(3, 2) portfolio sorted on (size, book-to-market)
74	3_3	Kenneth French	*lv*	(3, 3) portfolio sorted on (size, book-to-market)
75	3_4	Kenneth French	*lv*	(3, 4) portfolio sorted on (size, book-to-market)
76	3_5	Kenneth French	*lv*	(3, 5) portfolio sorted on (size, book-to-market)
77	3_6	Kenneth French	*lv*	(3, 6) portfolio sorted on (size, book-to-market)
78	3_7	Kenneth French	*lv*	(3, 7) portfolio sorted on (size, book-to-market)
79	3_8	Kenneth French	*lv*	(3, 8) portfolio sorted on (size, book-to-market)
80	3_9	Kenneth French	*lv*	(3, 9) portfolio sorted on (size, book-to-market)
81	3_high	Kenneth French	*lv*	(3, high) portfolio sorted on (size, book-to-market)
82	4_low	Kenneth French	*lv*	(4, low) portfolio sorted on (size, book-to-market)
83	4_2	Kenneth French	*lv*	(4, 2) portfolio sorted on (size, book-to-market)
84	4_3	Kenneth French	*lv*	(4, 3) portfolio sorted on (size, book-to-market)
85	4_4	Kenneth French	*lv*	(4, 4) portfolio sorted on (size, book-to-market)
86	4_5	Kenneth French	*lv*	(4, 5) portfolio sorted on (size, book-to-market)
87	4_6	Kenneth French	*lv*	(4, 6) portfolio sorted on (size, book-to-market)
88	4_7	Kenneth French	*lv*	(4, 7) portfolio sorted on (size, book-to-market)
89	4_8	Kenneth French	*lv*	(4, 8) portfolio sorted on (size, book-to-market)
90	4_9	Kenneth French	*lv*	(4, 9) portfolio sorted on (size, book-to-market)
91	4_high	Kenneth French	*lv*	(4, high) portfolio sorted on (size, book-to-market)
92	5_low	Kenneth French	*lv*	(5, low) portfolio sorted on (size, book-to-market)
93	5_2	Kenneth French	*lv*	(5, 2) portfolio sorted on (size, book-to-market)
94	5_3	Kenneth French	*lv*	(5, 3) portfolio sorted on (size, book-to-market)
95	5_4	Kenneth French	*lv*	(5, 4) portfolio sorted on (size, book-to-market)
96	5_5	Kenneth French	*lv*	(5, 5) portfolio sorted on (size, book-to-market)
97	5_6	Kenneth French	*lv*	(5, 6) portfolio sorted on (size, book-to-market)
98	5_7	Kenneth French	*lv*	(5, 7) portfolio sorted on (size, book-to-market)
99	5_8	Kenneth French	*lv*	(5, 8) portfolio sorted on (size, book-to-market)
100	5_9	Kenneth French	*lv*	(5, 9) portfolio sorted on (size, book-to-market)
101	5_high	Kenneth French	*lv*	(5, high) portfolio sorted on (size, book-to-market)
102	6_low	Kenneth French	*lv*	(6, low) portfolio sorted on (size, book-to-market)
103	6_2	Kenneth French	*lv*	(6, 2) portfolio sorted on (size, book-to-market)
104	6_3	Kenneth French	*lv*	(6, 3) portfolio sorted on (size, book-to-market)
105	6_4	Kenneth French	*lv*	(6, 4) portfolio sorted on (size, book-to-market)
106	6_5	Kenneth French	*lv*	(6, 5) portfolio sorted on (size, book-to-market)
107	6_6	Kenneth French	*lv*	(6, 6) portfolio sorted on (size, book-to-market)
108	6_7	Kenneth French	*lv*	(6, 7) portfolio sorted on (size, book-to-market)
109	6_8	Kenneth French	*lv*	(6, 8) portfolio sorted on (size, book-to-market)
110	6_9	Kenneth French	*lv*	(6, 9) portfolio sorted on (size, book-to-market)
111	6_high	Kenneth French	*lv*	(6, high) portfolio sorted on (size, book-to-market)
112	7_low	Kenneth French	*lv*	(7, low) portfolio sorted on (size, book-to-market)
113	7_2	Kenneth French	*lv*	(7, 2) portfolio sorted on (size, book-to-market)
114	7_3	Kenneth French	*lv*	(7, 3) portfolio sorted on (size, book-to-market)
115	7_4	Kenneth French	*lv*	(7, 4) portfolio sorted on (size, book-to-market)
116	7_5	Kenneth French	*lv*	(7, 5) portfolio sorted on (size, book-to-market)
117	7_6	Kenneth French	*lv*	(7, 6) portfolio sorted on (size, book-to-market)

No.	Short Name	Source	Tran	Description
118	7_7	Kenneth French	lv	(7, 7) portfolio sorted on (size, book-to-market)
119	7_8	Kenneth French	lv	(7, 8) portfolio sorted on (size, book-to-market)
120	7_9	Kenneth French	lv	(7, 9) portfolio sorted on (size, book-to-market)
121	8_low	Kenneth French	lv	(8, low) portfolio sorted on (size, book-to-market)
122	8_2	Kenneth French	lv	(8, 2) portfolio sorted on (size, book-to-market)
123	8_3	Kenneth French	lv	(8, 3) portfolio sorted on (size, book-to-market)
124	8_4	Kenneth French	lv	(8, 4) portfolio sorted on (size, book-to-market)
125	8_5	Kenneth French	lv	(8, 5) portfolio sorted on (size, book-to-market)
126	8_6	Kenneth French	lv	(8, 6) portfolio sorted on (size, book-to-market)
127	8_7	Kenneth French	lv	(8, 7) portfolio sorted on (size, book-to-market)
128	8_8	Kenneth French	lv	(8, 8) portfolio sorted on (size, book-to-market)
129	8_9	Kenneth French	lv	(8, 9) portfolio sorted on (size, book-to-market)
130	8_high	Kenneth French	lv	(8, high) portfolio sorted on (size, book-to-market)
131	9_low	Kenneth French	lv	(9, low) portfolio sorted on (size, book-to-market)
132	9_2	Kenneth French	lv	(9, 2) portfolio sorted on (size, book-to-market)
133	9_3	Kenneth French	lv	(9, 3) portfolio sorted on (size, book-to-market)
134	9_4	Kenneth French	lv	(9, 4) portfolio sorted on (size, book-to-market)
135	9_5	Kenneth French	lv	(9, 5) portfolio sorted on (size, book-to-market)
136	9_6	Kenneth French	lv	(9, 6) portfolio sorted on (size, book-to-market)
137	9_7	Kenneth French	lv	(9, 7) portfolio sorted on (size, book-to-market)
138	9_8	Kenneth French	lv	(9, 8) portfolio sorted on (size, book-to-market)
139	9_high	Kenneth French	lv	(9, high) portfolio sorted on (size, book-to-market)
140	10_low	Kenneth French	lv	(10, low) portfolio sorted on (size, book-to-market)
141	10_2	Kenneth French	lv	(10, 2) portfolio sorted on (size, book-to-market)
142	10_3	Kenneth French	lv	(10, 3) portfolio sorted on (size, book-to-market)
143	10_4	Kenneth French	lv	(10, 4) portfolio sorted on (size, book-to-market)
144	10_5	Kenneth French	lv	(10, 5) portfolio sorted on (size, book-to-market)
145	10_6	Kenneth French	lv	(10, 6) portfolio sorted on (size, book-to-market)
146	10_7	Kenneth French	lv	(10, 7) portfolio sorted on (size, book-to-market)
147	VXO	Fred MD	lv	VXOCLS

CRSP Data Details Value-weighted price and dividend data were obtained from the Center for Research in Security Prices (CRSP, Center for Research in Security Prices (1926–2022)). From the Annual Update data, we obtain the monthly value-weighted return series `vwretd` (with dividends) and `vwretx` (excluding dividends). These series have the interpretations: $VWRET_t = \frac{P_{t+1}+D_{t+1}}{P_t}$, $VWRETX_t = \frac{P_{t+1}}{P_t}$. From these series, a normalized price series P_t can be constructed recursively as: $P_0 = 1$, $P_t = P_{t-1} \times VWRETX_{t-1}$. A dividend series can then be constructed using: $D_t = P_{t-1} \times (VWRET_{t-1} - VWRETX_{t-1})$. In order to remove seasonality of dividend payments from the data, instead of D_t we use the series: $\bar{D}_t = \frac{1}{12}\sum_{j=0}^{11} D_{t-j}$, i.e., the moving average over the entire year. For the price and dividend series under "reinvestment," we calculate the price under reinvestment, P_t^{re}, as the normalized value of the market portfolio under reinvestment of dividends, using the recursion: $P_0^{re} = 1$, $P_t^{re} = P_{t-1} \times VWRET_{t-1}$. Similarly, we can define dividends under reinvestment, D_t^{re}, as the total dividend payments on this portfolio (the number of "shares" of which have increased over time) using: $D_t^{re} = P_{t-1}^{re} \times (VWRET_{t-1} - VWRETX_{t-1})$. As before, we can remove seasonality by using: $\bar{D}_t^{re} = \frac{1}{12}\sum_{j=0}^{11} D_{t-j}^{re}$. Five data series are constructed from the CRSP data as follows: `D_log(DIV)`: $\Delta\log(\bar{D}_t)$; `D_log(P)`: $\Delta\log(P_t)$; `D_DIVreinvest`: $\Delta\log(\bar{D}_t^{re})$; `D_Preinvest`: $\Delta\log(P_t^{re})$; `d-p`: $\log(\bar{D}_t) - \log(P_t)$.

Kenneth French Data Details The following data are obtained from the data library of Kenneth French's Dartmouth website (French (1926–2022)):

- Fama/French Factors: From this dataset we obtain the series `RF`, `Mkt-RF`, `SMB`, and `HML`.
- 25 Portfolios Formed on Size and Book-to-Market (5 x 5): From this dataset we obtain the series `R15-R11`, which is the return spread between the (small, high book-to-market) and (small, low book-to-market) portfolios.
- Momentum Factor (Mom): From this dataset we obtain the series `UMD`, which is equal to the momentum factor.
- 49 Industry Portfolios: From this dataset we use all value-weighted series, excluding any series that have missing observations from January 1960 onward. This yields the series `Agric` through `Other`. The omitted series are `Soda`, `Hlth`, `FabPr`, `Guns`, `Gold`, and `Softw`.
- 100 Portfolios Formed on Size and Book-to-Market: From this dataset we use all value-weighted series, excluding any series that have missing observations from January 1960 onward. This yields variables with names `X_Y`, where X denotes the size index $(1, 2, \ldots, 10)$ and Y denotes the book-to-market index $(\text{Low}, 2, 3, \ldots, 8, 9, \text{High})$. The omitted series are `1_low`, `1_3`, `7_high`, `9_9`, `10_8`, `10_9`, and `10_high`.

VXO Data Details VXO data is obtained from the Monthly Database for Macroeconomic Research (FRED-MD, McCracken (2015–2022)).

OE.2.6 Daily Financial Data

Daily Data and construction of daily factors These data are used in the machine learning forecasts. The daily financial series in this data set are from the daily financial dataset used in Andreou et al. (2013). I create a smaller daily database which is a subset of the large cross-section of 991 daily series in their dataset. Our dataset covers five classes of financial assets: (i) the Commodities class; (ii) the Corporate Risk category; (iii) the Equities class; (iv) the Foreign Exchange Rates class and (v) the Government Securities. The dataset includes up to 87 daily predictors in a daily frequency from 23-Oct-1959 to 31-Dec-2023 from the above five categories of financial assets. I remove series with fewer than ten years of data and time periods with no variables observed, which occurs for some series in the early part of the sample. For those years, I have less than 87 series. There are 39 commodity variables which include commodity indices, prices and futures, 16 corporate risk series, 9 equity series which include major US stock market indices and the 500 Implied Volatility, 16 government securities which include the federal funds rate, government treasury bills of securities from three months to ten years, and 7 foreign exchange variables which include the individual foreign exchange rates of major five US trading partners and two effective exchange rate. I choose these daily predictors because they are proposed in the literature as good predictors of economic growth.

I construct daily financial factors in a quarterly frequency in two steps. First, I use these daily financial time series to form factors at a daily frequency. The raw data used to form factors are always transformed to achieve stationarity and standardized before performing factor estimation (see generic description below). I re-estimate factors at each date in the sample recursively over time using the entire history of data available in real time prior to each out-of-sample forecast. In the second step, I convert these daily financial indicators to quarterly weighted variables to form quarterly factors by selecting an optimal weighting scheme according to the method described below (see the weighting scheme section). The data series used in this dataset are listed below in Table OA.15 by data source. The tables also list the transformation applied to each variable to make them stationary before generating factors. The transformations used to stationarize a time series are the same as those explained in the section "Monthly financial factor data".

Table OA.15: List of Daily Financial Dataset Variables

No.	Short Name	Source	Tran	Description
			Group 1: Commodities	
1	GSIZSPT	Data Stream	Δln	S&P GSCI Zinc Spot - PRICE INDEX
2	GSSBSPT	Data Stream	Δln	S&P GSCI Sugar Spot - PRICE INDEX
3	GSSOSPT	Data Stream	Δln	S&P GSCI Soybeans Spot - PRICE INDEX
4	GSSISPT	Data Stream	Δln	S&P GSCI Silver Spot - PRICE INDEX
5	GSIKSPT	Data Stream	Δln	S&P GSCI Nickel Spot - PRICE INDEX
6	GSLCSPT	Data Stream	Δln	S&P GSCI Live Cattle Spot - PRICE INDEX
7	GSLHSPT	Data Stream	Δln	S&P GSCI Lean Hogs Index Spot - PRICE INDEX
8	GSILSPT	Data Stream	Δln	S&P GSCI Lead Spot - PRICE INDEX
9	GSGCSPT	Data Stream	Δln	S&P GSCI Gold Spot - PRICE INDEX
10	GSCTSPT	Data Stream	Δln	S&P GSCI Cotton Spot - PRICE INDEX
11	GSKCSPT	Data Stream	Δln	S&P GSCI Coffee Spot - PRICE INDEX
12	GSCCSPT	Data Stream	Δln	S&P GSCI Cocoa Index Spot - PRICE INDEX
13	GSIASPT	Data Stream	Δln	S&P GSCI Aluminum Spot - PRICE INDEX
14	SGWTSPT	Data Stream	Δln	S&P GSCI All Wheat Spot - PRICE INDEX
15	EIAEBRT	Data Stream	Δln	Europe Brent Spot FOB U$/BBL Daily
16	CRUDOIL	Data Stream	Δln	Crude Oil-WTI Spot Cushing U$/BBL - MID PRICE
17	LTICASH	Data Stream	Δln	LME-Tin 99.85% Cash U$/MT
18	CWFCS00	Data Stream	Δln	CBT-WHEAT COMPOSITE FUTURES CONT. - SETT. PRICE
19	CCFCS00	Data Stream	Δln	CBT-CORN COMP. CONTINUOUS - SETT. PRICE
20	CSYCS00	Data Stream	Δln	CBT-SOYBEANS COMP. CONT. - SETT. PRICE
21	NCTCS20	Data Stream	Δln	CSCE-COTTON #2 CONT.2ND FUT - SETT. PRICE
22	NSBCS00	Data Stream	Δln	CSCE-SUGAR #11 CONTINUOUS - SETT. PRICE
23	NKCCS00	Data Stream	Δln	CSCE-COFFEE C CONTINUOUS - SETT. PRICE
24	NCCCS00	Data Stream	Δln	CSCE-COCOA CONTINUOUS - SETT. PRICE
25	CZLCS00	Data Stream	Δln	ECBOT-SOYBEAN OIL CONTINUOUS - SETT. PRICE
26	COFC01	Data Stream	Δln	CBT-OATS COMP. TRc1 - SETT. PRICE
27	CLDCS00	Data Stream	Δln	CME-LIVE CATTLE COMP. CONTINUOUS - SETT. PRICE
28	CLGC01	Data Stream	Δln	CME-LEAN HOGS COMP. TRc1 - SETT. PRICE
29	NGCCS00	Data Stream	Δln	CMX-GOLD 100 OZ CONTINUOUS - SETT. PRICE
30	LAH3MTH	Data Stream	Δln	LME-Aluminium 99.7% 3 Months U$/MT
31	LED3MTH	Data Stream	Δln	LME-Lead 3 Months U$/MT
32	LNI3MTH	Data Stream	Δln	LME-Nickel 3 Months U$/MT
33	LTI3MTH	Data Stream	Δln	LME-Tin 99.85% 3 Months U$/MT
34	PLNYD	www.macrotrends.net	Δln	Platinum Cash Price (U$ per troy ounce)
35	XPDD	www.macrotrends.net	Δln	Palladium (U$ per troy ounce)
36	CUS2D	www.macrotrends.net	Δln	Corn Spot Price (U$/Bushel)
37	SoybOil	www.macrotrends.net	Δln	Soybean Oil Price (U$/Pound)

No.	Short Name	Source	Tran	Description
38	OATSD	www.macrotrends.net	Δln	Oat Spot Price (US$/Bushel)
39	WTIOilFut	US EIA	Δln	Light Sweet Crude Oil Futures Price: 1St Expiring Contract Settlement ($/Bbl)
		Group 2: Equities		
40	S&PCOMP	Data Stream	Δln	S&P 500 COMPOSITE - PRICE INDEX
41	ISPCS00	Data Stream	Δln	CME-S&P 500 INDEX CONTINUOUS - SETT. PRICE
42	SP5EIND	Data Stream	Δln	S&P500 ES INDUSTRIALS - PRICE INDEX
43	DJINDUS	Data Stream	Δln	DOW JONES INDUSTRIALS - PRICE INDEX
44	CYMCS00	Data Stream	Δln	CBT-MINI DOW JONES CONTINUOUS - SETT. PRICE
45	NASCOMP	Data Stream	Δln	NASDAQ COMPOSITE - PRICE INDEX
46	NASA100	Data Stream	Δln	NASDAQ 100 - PRICE INDEX
47	CBOEVIX	Data Stream	lv	CBOE SPX VOLATILITY VIX (NEW) - PRICE INDEX
48	S&P500toVIX	Data Stream	Δln	S&P500/VIX
		Group 3: Corporate Risk		
49	LIBOR	FRED	Δlv	Overnight London Interbank Offered Rate (%)
50	1MLIBOR	FRED	Δlv	1-Month London Interbank Offered Rate (%)
51	3MLIBOR	FRED	Δlv	3-Month London Interbank Offered Rate (%)
52	6MLIBOR	FRED	Δlv	6-Month London Interbank Offered Rate (%)
53	1YLIBOR	FRED	Δlv	One-Year London Interbank Offered Rate (%)
54	1MEuro-FF	FRED	lv	1-Month Eurodollar Deposits (London Bid) (% P.A.) minus Fed Funds
55	3MEuro-FF	FRED	lv	3-Month Eurodollar Deposits (London Bid) (% P.A.) minus Fed Funds
56	6MEuro-FF	FRED	lv	6-Month Eurodollar Deposits (London Bid) (% P.A.) minus Fed Funds
57	APFNF-AANF	Data Stream	lv	1-Month A2/P2/F2 Nonfinancial Commercial Paper (NCP) (% P. A.) minus 1-Month Aa NCP (% P.A.)
58	APFNF-AAF	Data Stream	lv	1-Month A2/P2/F2 NCP (% P.A.) minus 1-Month Aa Financial Commercial Paper (% P.A.)
59	TED	Data Stream, FRED	lv	3Month Tbill minus 3-Month London Interbank Offered Rate (%)
60	MAaa-10YTB	Data Stream	lv	Moody Seasoned Aaa Corporate Bond Yield (% P.A.) minus Y10-Tbond
61	MBaa-10YTB	Data Stream	lv	Moody Seasoned Baa Corporate Bond Yield (% P.A.) minus Y10-Tbond
62	MLA-10YTB	Data Stream, FRED	lv	Merrill Lynch Corporate Bonds: A Rated: Effective Yield (%) minus Y10-Tbond
63	MLAA-10YTB	Data Stream, FRED	lv	Merrill Lynch Corporate Bonds: Aa Rated: Effective Yield (%) minus Y10-Tbond
64	MLAAA-10YTB	Data Stream, FRED	lv	Merrill Lynch Corporate Bonds: Aaa Rated: Effective Yield (%) minus Y10-Tbond
		Group 4: Treasuries		
65	FRFEDFD	Data Stream	Δlv	US FED FUNDS EFF RATE (D) - MIDDLE RATE
66	FRTBS3M	Data Stream	Δlv	US T-BILL SEC MARKET 3 MONTH (D) - MIDDLE RATE
67	FRTBS6M	Data Stream	Δlv	US T-BILL SEC MARKET 6 MONTH (D) - MIDDLE RATE
68	FRTCM1Y	Data Stream	Δlv	US TREASURY CONST MAT 1 YEAR (D) - MIDDLE RATE
69	FRTCM10	Data Stream	Δlv	US TREASURY CONST MAT 10 YEAR (D) - MIDDLE RATE
70	6MTB-FF	Data Stream	lv	6-month treasury bill market bid yield at constant maturity (%) minus Fed Funds
71	1YTB-FF	Data Stream	lv	1-year treasury bill yield at constant maturity (% P.A.) minus Fed Funds
72	10YTB-FF	Data Stream	lv	10-year treasury bond yield at constant maturity (% P.A.) minus Fed Funds
73	6MTB-3MTB	Data Stream	lv	6-month treasury bill yield at constant maturity (% P.A.) minus 3M-Tbills
74	1YTB-3MTB	Data Stream	lv	1-year treasury bill yield at constant maturity (% P.A.) minus 3M-Tbills
75	10YTB-3MTB	Data Stream	lv	10-year treasury bond yield at constant maturity (% P.A.) minus 3M-Tbills
76	BKEVEN05	FRB	lv	US Inflation compensation: continuously compounded zero-coupon yield: 5-year (%)
77	BKEVEN10	FRB	lv	US Inflation compensation: continuously compounded zero-coupon yield: 10-year (%)
78	BKEVEN1F4	FRB	lv	BKEVEN1F4
79	BKEVEN1F9	FRB	lv	BKEVEN1F9
80	BKEVEN5F5	FRB	lv	US Inflation compensation: coupon equivalent forward rate: 5-10 years (%)
		Group 5: Foreign Exchange (FX)		
81	US_CWBN	Data Stream	Δln	US NOMINAL DOLLAR BROAD INDEX - EXCHANGE INDEX
82	US_CWMN	Data Stream	Δln	US NOMINAL DOLLAR MAJOR CURR INDEX - EXCHANGE INDEX

No.	Short Name	Source	Tran	Description
83	US_CSFR2	Data Stream	Δln	CANADIAN $ TO US $ NOON NY - EXCHANGE RATE
84	EU_USFR2	Data Stream	Δln	EURO TO US$ NOON NY - EXCHANGE RATE
85	US_YFR2	Data Stream	Δln	JAPANESE YEN TO US $ NOON NY - EXCHANGE RATE
86	US_SFFR2	Data Stream	Δln	SWISS FRANC TO US $ NOON NY - EXCHANGE RATE
87	US_UKFR2	Data Stream	Δln	UK POUND TO US $ NOON NY - EXCHANGE RATE

From Daily to Quarterly Factors: Weighting Schemes After we obtain daily financial factors $G_{D,t}$, we use weighting schemes proposed in the literature on Mixed Data Sampling (MIDAS) regressions to form quarterly factors, denoted $G_{D,t}^{Q}$. Let G_t^D denote a factor in daily frequency formed from the daily financial dataset, and let G_t^Q denote a quarterly aggregate of the corresponding daily factor time series. Let $G^D_{ND-j,d_t,t}$ denote the value of a daily factor on the j-th day counting backwards from the survey deadline d_t in quarter t. Hence, the day d_t of quarter t corresponds to $j = 0$, so the daily factor on the survey deadline is $G^D_{ND,d_t,t}$. For simplicity, we suppress the subscript d_t, writing $G^D_{ND-j,t}$.

We compute the quarterly aggregate of a daily financial factor as a weighted average of observations over the ND business days before the survey deadline. This means that the forecaster's information set includes daily financial data up to the previous ND business days before the survey deadline. The quarterly factor G_t^Q is defined as:

$$G_t^Q(w) = \sum_{j=1}^{ND} w_j \times G^D_{ND-j,t}$$

where w_j is a weight. We consider the following three types of weighting schemes to convert daily factor observations to quarterly aggregates. Each weighting scheme weights information by some function of the number of days prior to the survey deadline.

1. $w_i = 1$ for $i = 1$ and $w_i = 0$ otherwise. This weighting scheme places all weight on the data from the last business day before the survey deadline and zero weight on any data prior to that day.

2. $w_i = \delta^i \big/ \sum_{j=1}^{ND} \delta^j$, where we consider a range of δ values with $\delta \in \{0.1, 0.2, 0.3, 0.7, 0.8, 0.9, 1.0\}$. The smaller the δ, the more rapidly information prior to the survey deadline is down-weighted. This down-weighting is progressive but not non-monotonic. The case $\delta = 1$ corresponds to a simple average of observations across all days.

3. The third parameterization uses two parameters $\theta = (\theta_1, \theta_2)'$ and allows for non-monotonic weighting of past information. The weights are defined as:

$$w(i; \theta_1, \theta_2) = \frac{f\left(\frac{i}{ND}; \theta_1, \theta_2\right)}{\sum_{j=1}^{ND} f\left(\frac{j}{ND}; \theta_1, \theta_2\right)}$$

   where $f(x; a, b) = x^{a-1}(1-x)^{b-1} \cdot \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}$, and $\Gamma(a)$ is the gamma function $\Gamma(a) = \int_0^\infty x^{a-1}e^{-x}\,dx$. The weights $w(i; \theta_1, \theta_2)$ are the Beta polynomial MIDAS weights of Ghysels et al. (2007), based on the Beta function. This weighting scheme is flexible enough to generate a wide range of possible shapes with only two parameters.

We consider these possible weighting schemes and choose the optimal weighting scheme w^* from 24 candidate weighting schemes for each daily financial factor G_t^D by minimizing the sum of squared residuals in a regression of $y_{j,t+h}$ on G_t^Q:

$$y_{j,t+h} = \alpha + \beta \times G_t^Q(w) + u_{t+h}$$

This procedure is conducted in real time using recursive regressions. We re-estimate the weights at each date in the sample recursively over time, using the entire history of data available in real time prior to each out-of-sample forecast. We assume that $ND = 14$, which implies that forecasters use daily information from at most the past two weeks before the survey deadline. This process is repeated for each daily financial factor in $G_{D,t}$ to form quarterly factors $G_{D,t}^Q$.

OE.2.7 LDA Data

The LDA data are used as inputs into the machine learning forecasts. The database for our Latent Dirichlet Allocation (LDA) analysis contains around one million articles published in *Wall Street Journal* between January 1984 to Dec 2023. The current vintage of the results reported here is based a randomly selected sub-sample of 200,000 articles over the same period, one-fifth size of the entire database. The sample selection procedure follows Bybee et al. (2021). First, I remove all articles prior to January 1984 and after June 2022 and exclude articles published in weekends. Second, I exclude articles with subject tags associated with obviously non-economic content such as sports. Third, I exclude articles with the certain headline patterns, such as those associated with data tables or those corresponding to regular sports, leisure, or books columns. I filter the articles using the same list of exclusions provided by Bybee et al. (2021). Last, I exclude articles with less than 100 words.

Processing of texts The processing of the texts can be summarized into five steps:

1. Tokenization: parse each article's text into a white-space-separated word list retaining the article's word ordering.

2. I drop all non-alphabetical characters and set the remaining characters to lower-case, remove words with less than 3 letters, and remove common stop words and URL-based terms. I use a standard list of stop words from the Python library *gensim.parsing.preprocessing.*

3. Lemmatization and Stemming: lemmatization returns the original form of a word using external dictionary *Textblob.Word* in Python and based on the context of the word. For instance, as a verb, "went" is converted to"go". Stemming usually refers to a heuristic process that removes the trailing letters at the end of the words, such as from "assesses" to "assess', and "really" to "real". I use the Python library *Textblob.Word* to implement the lemmatization and *SnowballStemmer* for the stemming. The results are not very sensitive to the particular Python packages being used.

4. From the first three steps, I obtain a list of uni-grams which are a list of singular words. For example, "united" and "states" are uni-grams from "united states". From the list of uni-grams, I generate a set of bi-grams as all pairs of (ordered) adjacent uni-grams. For example, "united states" together is one bi-gram. I then exclude uni-grams and bi-grams appearing in less than 0.1% of articles.

5. Last, I convert an article's word list into a vector of counts for each uni-gram and bi-gram. For example, the vector of counts $[5, 7, 2]$ corresponds to the number of times the words $[\text{"}federal\text{"}, \text{"}reserve\text{"}, \text{"}bank\text{"}]$ appear in the article.

The LDA Model The LDA model Blei et al. (2003) essentially achieves substantial dimension reduction of the word distribution of each article using the following assumptions. I assume a factor structure on the vectors of word counts. Each factor is a topic and each article is a parametric distribution of topics, specified as follows,

$$\underbrace{\overbrace{w_i}^{V\times 1}}_{\text{word dist of article } i} \sim \text{Mult}\left(\underbrace{\overbrace{\Phi'}^{V\times K}}_{\text{topic-word dist.}}, \underbrace{\overbrace{\theta_i}^{K\times 1}}_{\text{topic dist.}}, \underbrace{N_i}_{\text{\# of words}}\right)$$

where Mult is the multinomial distribution. In the above equation, w_i is a vector of word counts of each unique term (uni-gram or bi-gram) in article i, whose size is equal to the number of unique terms V. K is the number of factors in article i. In the estimation, I assume $K = 180$ following Bybee et al. (2021). Φ is a matrix sized $K \times V$, whose kth row and vth column is equal to the probability of the unique term v showing up in topic k. θ_i stores the weights of all k topics contained in article i, which sum up to one. Dimension reduction is achieved as long as $K << V$ (the number of topics are significantly smaller than the number of unique terms). More specifically, it reduces the dimension from $T \times V$ to $T \times K$ (the size of θ) + $K \times V$ (the size of Φ).

Real-time news factors. I also generate real-time news factors for each month t starting from January 1991. In theory, I could train the LDA model using each real-time monthly vintage but it is computationally challenging. Instead, I simplify the procedure by training the LDA model using quarterly vintages t, $t + 3$, $t + 6$, etc, and use the LDA model parameters estimated at t to filter news paper articles within the quarter and generate news factors for those months. More specifically, given every article's word distribution $w_{i,t+s}$,for $s = 0, 1, 2$, and the estimated real-time topic-word distribution parameters $\hat{\Phi}_t$ using articles till date t, one can obtain the filtered topic distribution of each article $\hat{\theta}_{i,t+s}$, as follows,

$$\underbrace{\overbrace{w_{i,t+s}}^{V\times 1}}_{\text{word dist of article } i \text{ at time } t+s} \sim \text{Mult}\left(\underbrace{\overbrace{\hat{\Phi}'}^{V\times K}}_{\text{topic-word dist.}}, \underbrace{\overbrace{\hat{\theta}_{i,t+s}}^{K\times 1}}_{\text{topic dist.}}, \underbrace{N_{i,t+s}}_{\text{\# of words}}\right).$$

LDA Estimation I use the built-in LDA model estimation toolbox in the Python library `https://pypi.org/project/gensim/`Gensim to implement the model estimation. The model requires following initial inputs and parameters and it is estimated using Bayesian methods. In theory, maximum-likelihood estimation is possible but it is computationally challenging.

1. I create a document-term matrix $\mathbf{W}$ as a collection of w_i for all articles i in the sample. The number of rows in $\mathbf{W}$ is equal to the number of articles in our sample and the number of columns in $\mathbf{W}$ is equal to the number of unique uni-gram and bi-grams (after being filtered) across all articles. The matrix $\mathbf{W}$ is used as an input for the LDA model estimation. I then follow Bybee et al. (2021) and set the number of topics K to be 180. The authors used Bayesian criteria to find 180 to be an optimal number of topics.

2. In the Python library Gensim, the key parameters of the LDA estimation are α and β. With a higher value of α, the documents are composed of more topics. With a higher value of β, each topic contains more terms (uni- or bi-grams). In the implementations, I do not impose any explicit restrictions on initial values of those parameters and set them to be "auto". These two parameters, alongside Φ' and $\{\theta_i\}_i$, are estimated by the toolbox from Python library `https://pypi.org/project/gensim/`Gensim.

Real-time LDA Factors With the estimated topic weights $\theta_{i,t}$ of each article i from the LDA model, I further construct time series of the overall news attention to each topic, or a news factor. The value of the topic k at time t is the average weights of topic k of all articles published at t, specified as follows,

$$F_{k,t} = \frac{\sum_i \hat{\theta}_{i,k,t}}{\#\text{ of articles at } t}$$

for all topics k. We construct daily LDA factors by aggregating all articles published on each calendar day. The value of topic k at day t is the average weights of topic k across all articles published that day.

OE.2.8 Machine Variables to Be forecast

Returns and price growth When evaluating the MSE ratio of the machine relative to that of a benchmark survey, we use the machine forecast for the return or price growth measure that most closely corresponds to the concept that survey respondents are asked to predict:

1. CFO survey asks respondents about their expectations for the S&P 500 return over the next 12 months. Following Nagel and Xu (2021), we interpret the survey to be asking about $r^d_{t,t+12}$, the one-year CRSP value-weighted return (including dividends) from the current survey month to the same month one year ahead.

2. Gallup/UBS survey respondents report the return (including dividends) they expect on their own portfolio one year ahead. We interpret the survey to be asking about $r^d_{t,t+12}$, the one-year CRSP value-weighted return(including dividends) from the current survey month to the same month one year ahead.

3. Livingston survey respondents provide 12-month ahead forecasts of the S&P 500 index. We convert the level forecast to price growth forecast by taking the log difference between the 12-month ahead level forecast and the nowcast of the S&P 500 index for the current survey month. Therefore, we interpret the survey to be asking about the one-year price growth in the S&P 500 index.

4. Bloomberg Consensus Forecasts asks survey respondents about the end-of-year closing value of the S&P 500 index. We interpret the survey to be asking about the h-month price growth in the S&P 500 index. The horizon of the forecast changes depending on when in the year the panelists are answering the survey.

5. Michigan Survey of Consumers (SOC) asks respondents about their perceived probability that an investment in a diversified stock fund would increase in value in the year ahead. We interpret the question to be asking about the one-year price growth in the S&P 500 index.

6. Conference Board (CB) survey asks respondents about their categorical belief on whether they expect stock prices to increase, decrease, or stay the same over the next year. We interpret the question to be asking about the one-year price growth in the S&P 500 index.

Earnings growth (IBES "Street" Earnings) For earnings growth forecasts, we use a quarterly S&P 500 total earnings series based on IBES street earnings per share (EPS), as described above. Street earnings exclude discontinued operations, extraordinary charges, and other non-operating items, making them better aligned with the earnings measure targeted by survey respondents. We convert EPS to total earnings using the S&P 500 index divisor and use the resulting quarterly series directly, prior to any monthly interpolation, since the machine learning algorithm operates at a quarterly frequency. The IBES street earnings series spans 1983Q4 to 2021Q4.

For Long-Term Growth (LTG) forecasts, IBES defines LTG as the "expected annual increase in operating earnings over the company's next full business cycle. These forecasts refer to a period of between three to five years." We compare survey responses of LTG against machine forecasts under alternative interpretations of LTG. First, we consider machine forecasts of annual five-year forward growth, i.e., annual earnings growth from four to five years ahead (Bianchi et al. (2024)). Second, we consider machine forecasts of annualized 5-year growth, i.e., annual earnings growth from current quarter to five years ahead, following the interpretation in Bordalo et al. (2019). Third, we consider machine forecasts of annualized earnings growth from one to 10 years ahead, following the interpretation in Nagel and Xu (2021)

Inflation We construct forecasts of annual inflation defined as $\pi_{t+4,t} = \log\left(\frac{PGDP_{t+4}}{PGDP_t}\right)$, where $PGDP_t$ is the quarterly level of the chain-weighted GDP price index. Following Coibion and Gorodnichenko (2015), we use the vintage of inflation data that is available four quarters after the period being forecast.

OE.2.9 Machine Input Data: Predictor Variables

The vector $\mathbf{Z}_{jt} \equiv \left(y_{j,t}, \hat{\mathbf{G}}_t', \mathbf{W}_{jt}'\right)'$ is an $r = 1 + r_G + r_W$ vector which collects the data at time t with

$$\mathcal{Z}_{jt} \equiv \left(y_{j,t}, ..., y_{j,t-p_y}, \hat{\mathbf{G}}_t', ..., \hat{\mathbf{G}}_{t-p_G}', \mathbf{W}_{jt}', ..., \mathbf{W}_{jt-p_W}'\right)'$$

a vector of contemporaneous and lagged values of $\mathbf{Z}_{jt}$, where p_y, p_G, p_W denote the total number of lags of $y_{j,t}$, $\hat{\mathbf{G}}_t'$, $\mathbf{W}_{jt}'$, respectively. The predictors below are listed as elements of $y_{j,t}, \hat{\mathbf{G}}_{jt}'$, or $\mathbf{W}_{jt}'$ for variables.

Stock return and price growth predictor variables and specifications For y_j equal to CRSP value-weighted returns or S&P 500 price index growth, we first predict the one-year log stock return or price growth that is expected to occur h quarters into the future from time $t+h-4$ to $t+h$, i.e., $\mathbb{E}_t[r_{t+h-4,t+h}]$. For horizons longer than one year, since the h-quarter long horizon return is the sum of one-year returns between time t to $t+h$, we first forecast the forward one-year returns separately and then add the components together to get machine forecasts of h-quarter long horizon returns. The forecasting model considers the following variables. Lags of the dependent variable:

1. y_{t-1}, y_{t-2} one and two quarter lagged stock returns or price growth.

The factors in $\hat{\mathbf{G}}_{jt}'$ are formed from three large datasets separately:

1. $\mathbf{G}_{M,t-k}$, for $k = 0, 1$ are factors formed from a real-time macro dataset $\mathcal{D}^M$ with 92 real-time macro series; includes both monthly and quarterly series, with monthly series converted to quarterly according to the method described in the data appendix.

2. $\mathbf{G}_{F,t-k}$, for $k = 0, 1$ are factors formed from a financial data set $\mathcal{D}^F$ with 147 monthly financial series.

3. $\mathbf{G}_{D,t-k}^Q$, for $k = 0$ are quarterly factors formed from a daily financial dataset $\mathcal{D}^D$ of 87 daily financial indicators. The raw daily series are first converted to daily factors $\mathbf{G}_{D,t}(\mathbf{w})$ and the daily factors are aggregated up to quarterly observations $\mathbf{G}_{D,t}^Q(\mathbf{w})$ using a weighted average of daily factors, with the weights $\mathbf{w}$ dependent on two free parameters that are chosen to minimize the sum of squared residuals in a regression of $y_{j,t+h}$ on $\mathbf{G}_{D,t}(\mathbf{w})$.

The variables in $\mathbf{W}_{jt}'$ include:

1. *LDA topics* $F_{k,t-j}$, for topic $k = 1, 2, ...50$ and $j = 0, 1$. The value of the topic k at time t is the average weights of topic k of all articles published at t.

2. *Macro data surprises* from the money market survey. The macro news include, GDP growth (Q/Q percentage change), core CPI (Month/Month change), unemployment rate (percentage point), and nonfarm payroll (month/month change). We include first release, second release, and final release for GDP growth. This constitutes six macro data surprises per quarter.

3. *FOMC surprises* are defined as the changes in the current-month, 1, 2, 6, 12, and 24 month-ahead federal funds futures (FFF) contract rate and the changes in the 1, 2, 4, and 8 quarter-ahead Eurodollar (ED) futures contracts, from 10 minutes before to 20 minutes after each FOMC announcement. When benchmarking against a survey, we use the last FOMC meeting before the survey deadline to compute surprises. For surveys that do not have a clear deadline, we compute surprises using from the last FOMC in the first month of the quarter. When benchmarking against moving average, we use the last FOMC meeting before the end of the first month in each quarter to compute surprises. This leaves 10 FOMC surprise variables per quarter.

4. *Stock market jumps* are accumulated 30-minute window negative and positive jumps in the S&P 500 around news events over the previous quarter.

5. *Long-term growth of earnings*: 5-year growth of the SP500 earnings per share.

6. *Short rates.* When forecasting returns or price growth, the machine controls for the current nominal short rate, $\log(1 + 3MTB_t/100)$, imposing a unit coefficient. This is equivalent to forecasting the future return minus the current short rate.

The 92 macro series in $\mathcal{D}^M$ are selected to represent broad categories of macroeconomic time series. The majority of these are real activity measures: real output and income, employment and hours, consumer spending, housing starts, orders and unfilled orders, compensation and labor costs, and capacity utilization measures. The dataset also includes commodity and price indexes and a handful of bond and stock market indexes, and foreign exchange measures. The financial dataset $\mathcal{D}^f$ is an updated monthly version of the of 147 variables comprised solely of financial market time series used in Ludvigson and Ng (2007). These data include valuation ratios such as the dividend-price ratio and earnings-price ratio, growth rates of aggregate dividends and prices, default and term spreads, yields on corporate bonds of different ratings grades, yields on Treasuries and yield spreads, and a broad cross-section of industry, size, book-market, and momentum portfolio equity returns. A detailed description of the series is given in the Data Appendix of the online supplementary file at www.sydneyludvigson.com/s/ucc_data_appendix.pdf. The 87 daily financial indicators in $\mathcal{D}^D$ include daily time series on commodities spot prices and futures prices, aggregate stock market indexes, volatility indexes, credit spreads and yield spreads, and exchange rates.

Earning growth predictor variables and specifications For y_t equal to S&P 500 log earning growth, we construct a forecasted value for y_t, denoted $\hat{y}_{t|t-h}$, based on information known up to time t using the following variables. Lags of the dependent variable:

1. y_{t-1}, y_{t-2} one and two quarter lagged earnings growth.

The factors in $\hat{\mathbf{G}}'_{jt}$ are formed from three large datasets separately:

1. $\mathbf{G}_{M,t-k}$, for $k = 0, 1$ are factors formed from a real-time macro dataset $\mathcal{D}^M$ with 92 real-time macro series; includes both monthly and quarterly series, with monthly series converted to quarterly according to the method described in the data appendix.

2. $\mathbf{G}_{F,t-k}$, for $k = 0, 1$ are factors formed from a financial data set $\mathcal{D}^F$ with 147 monthly financial series.

3. $\mathbf{G}^Q_{D,t-k}$, for $k = 0$ are quarterly factors formed from a daily financial dataset $\mathcal{D}^D$ of 87 daily financial indicators. The raw daily series are first converted to daily factors $\mathbf{G}_{D,t}(\mathbf{w})$ and the daily factors are aggregated up to quarterly observations $\mathbf{G}^Q_{D,t}(\mathbf{w})$ using a weighted average of daily factors, with the weights $\mathbf{w}$ dependent on two free parameters that are chosen to minimize the sum of squared residuals in a regression of $y_{j,t}$ on $\mathbf{G}_{D,t}(\mathbf{w})$.

The variables in $\mathbf{W}'_{jt}$ include:

1. *LDA factors* $F_{k,t-j}$, for topic $k = 1, 2, ...50$ and $j = 0, 1$. The value of the topic k at time t is the average weights of topic k of all articles published at t.

2. *Macro data surprises* from the money market survey. The macro news include, GDP growth (Q/Q percentage change), core CPI (Month/Month change), unemployment rate (percentage point), and nonfarm payroll (month/month change). We include first release, second release, and final release for GDP growth. This constitutes six macro data surprises per quarter.

3. *FOMC surprises* are defined as the changes in the current-month, 1, 2, 6, 12, and 24 month-ahead federal funds futures (FFF) contract rate and the changes in the 1, 2, 4, and 8 quarter-ahead Eurodollar (ED) futures contracts, from 10 minutes before to 20 minutes after each FOMC announcement. When benchmarking against a survey, we use the last FOMC meeting before the survey deadline to compute surprises. For surveys that do not have a clear deadline, we compute surprises using from the last FOMC in the first month of the quarter. When benchmarking against moving average, we use the last FOMC meeting before the end of the first month in each quarter to compute surprises. This leaves 10 FOMC surprise variables per quarter.

4. *Stock market jumps* are accumulated 30-minute window negative and positive jumps in the S&P 500 around news events over the previous quarter.

Inflation predictor variables For y_j equal to inflation, the forecasting model considers the following variables. Lags of the dependent variable:

1. $y_{t-1,t-h-1}$ one quarter lagged inflation.

The factors in $\hat{\mathbf{G}}'_{jt}$ are formed from three large datasets separately:

1. $\mathbf{G}_{M,t-k}$, for $k = 0, 1$ are factors formed from a real-time macro dataset $\mathcal{D}^M$ with 92 real-time macro series; includes both monthly and quarterly series, with monthly series converted to quarterly according to the method described in the data appendix.

2. $\mathbf{G}_{F,t-k}$, for $k = 0, 1$ are factors formed from a financial data set $\mathcal{D}^F$ with 147 monthly financial series.

3. $\mathbf{G}^Q_{D,t-k}$, for $k = 0$ are quarterly factors formed from a daily financial dataset $\mathcal{D}^D$ of 87 daily financial indicators. The raw daily series are first converted to daily factors $\mathbf{G}_{D,t}(\mathbf{w})$ and the daily factors are aggregated up to quarterly observations $\mathbf{G}^Q_{D,t}(\mathbf{w})$ using a weighted average of daily factors, with the weights $\mathbf{w}$ dependent on two free parameters that are chosen to minimize the sum of squared residuals in a regression of $y_{j,t+h}$ on $\mathbf{G}_{D,t}(\mathbf{w})$.

The variables in $\mathbf{W}'_{jt}$ include:

1. $\mathbb{F}^{(i)}_{jt-k}[y_{jt+h-k}]$, lagged values of the ith type's forecast, where $k = 1, 2$

2. $\mathbb{F}^{(s \neq i)}_{jt-1}[y_{jt+h-1}]$, lagged values of other type's forecasts, $s \neq i$

3. $var_N\left(\mathbb{F}^{(\cdot)}_{t-1}[y_{jt+h-1}]\right)$, where $var_N(\cdot)$ denotes the cross-sectional variance of lagged survey forecasts

4. $skew_N\left(\mathbb{F}^{(\cdot)}_{t-1}[y_{jt+h-1}]\right)$, where $skew_N(\cdot)$ denotes the cross-sectional skewness of lagged survey forecasts

5. Trend inflation measured as $\overline{\pi}_{t-1} = \begin{cases} \rho\overline{\pi}_{t-2} + (1-\rho)\pi_{t-1}, \rho = 0.95 & \text{if } t < \text{1991Q4} \\ \text{CPI10}_{t-1} & \text{if } t \geq \text{1991Q4} \end{cases}$, where CPI10 is the median SPF forecast of annualized average inflation over the current and next nine years. Trend inflation is intended to capture long-run trends. When long-run forecasts of inflation are not available, as is the case pre-1991Q4, we us a moving average of past inflation.

6. $\dot{GDP}_{t-1}$ = detrended gross domestic product, defined as the residual from a regression of GDP_{t-1} on a constant and the four most recent values of GDP as of date $t-8$. See Hamilton (2018).

7. $\dot{EMP}_{t-1}$ = detrended employment, defined as the residual from a regression of EMP_{t-1} on a constant and the four most recent values of EMP as of date $t-8$. See Hamilton (2018).

8. $\mathbb{N}^{(i)}_t[\pi_{t,t-h}]$ = Nowcast as of time t of the ith percentile of inflation over the period $t-h$ to t.

OE.3 Cross-Sectional Forecasts

I construct machine learning forecasts of stock returns and earnings growth at the firm level using the Long Short-Term Memory (LSTM) framework described in Section OE. The model is estimated using pooled panel data across all firms, with firm-specific predictors as inputs. I re-estimate model parameters and update hyperparameters every four quarters using a recursively expanding sample to maintain computational tractability while still incorporating new information. The stock universe consists of about 5,000 firms listed on the NYSE, AMEX, and NASDAQ with available IBES analyst coverage for one- and two-year ahead earnings expectations and long-term growth forecasts. Monthly total returns for these firms are obtained from CRSP. The sample spans March 1990 to December 2024.

To construct predictors, I follow the cross-sectional asset pricing literature and compile a broad set of stock-level characteristics. Specifically, I include 94 firm characteristics, of which 61 are updated annually, 13 quarterly, and 20 monthly. These characteristics span valuation ratios, profitability, investment, size, momentum, volatility, and other firm-level attributes, based on the definitions in Green et al. (2013). Book equity and operating profitability follow Fama and French (2015). I rank-transform each characteristic cross-sectionally within each month to the $[-1, 1]$ interval, as in Gu et al. (2020). I also include 74 industry dummies based on two-digit Standard Industrial Classification (SIC) codes. Table OA.16 provides further details on these predictors. To avoid forward-looking bias, I apply realistic reporting lags: monthly characteristics are assumed available with a one-month delay, quarterly characteristics with at least a four-month delay, and annual characteristics with at least a six-month delay. Missing values are replaced with the cross-sectional median at each period.

Following Gu et al. (2020), I construct an expanded set of predictors that interact firm-level characteristics with aggregate macroeconomic state variables. Let $\mathcal{C}_{i,t}$ denote the vector of firm characteristics for firm i, and let $\mathcal{X}_t$ denote the vector of aggregate predictors, which includes a constant and the same macroeconomic variables used to forecast aggregate returns, price growth, and earnings growth, respectively. The final predictor set for firm i at time t is given by $\mathcal{X}_{i,t} = \mathcal{X}_t \otimes \mathcal{C}_{i,t}$, where $\otimes$ denotes the Kronecker product. This structure generates interaction terms that capture how aggregate economic conditions influence the effect of firm-level characteristics on expected returns and earnings growth.

Table OA.16: Details of Firm Characteristics

No.	Acronym	Characteristic	Authors	Source	Frq.
1	absacc	Absolute accruals	Bandyopadhyay, Huang, Wirjanto 2010	Compustat	Y
2	acc	Working capital accruals	Sloan 1996	Compustat	Y
3	aeavol	Abnormal earnings ann volume	Lerman, Livnat, Mendenhall 2007	Compustat/CRSP	Q
4	age	Years since first coverage	Jiang, Lee, Zhang 2005	Compustat	Y
5	agr	Asset growth	Cooper, Gulen, Schill 2008	Compustat	Y
6	baspread	Bid-ask spread	Amihud, Mendelson 1989	CRSP	M
7	beta	Beta	Fama, MacBeth 1973	CRSP	M
8	betasq	Beta squared	Fama, MacBeth 1973	CRSP	M
9	bm	Book-to-market	Rosenberg, Reid, Lanstein 1985	Compustat/CRSP	Y
10	bm_ia	Industry-adj book-to-market	Asness, Porter, Stevens 2000	Compustat/CRSP	Y
11	cash	Cash holdings	Palazzo 2012	Compustat	Q
12	cashdebt	Cash flow to debt	Ou, Penman 1989	Compustat	Y
13	cashpr	Cash productivity	Chandrashekar, Rao 2009	Compustat	Y
14	cfp	Cash flow to price ratio	Desai, Rajgopal, Venkatachalam 2004	Compustat	Y
15	cfp_ia	Industry-adj cash flow to price ratio	Asness, Porter, Stevens 2000	Compustat	Y
16	chatoia	Industry-adj chg asset turnover	Soliman 2008	Compustat	Y
17	chcsho	Chg shares outstanding	Pontiff, Woodgate 2008	Compustat	Y
18	chempia	Industry-adj chg employees	Asness, Porter, Stevens 1994	Compustat	Y
19	chinv	Chg inventory	Thomas, Zhang 2002	Compustat	Y
20	chmom	Chg 6-month momentum	Gettleman, Marks 2006	CRSP	M
21	chpmia	Industry-adj chg profit margin	Soliman 2008	Compustat	Y
22	chtx	Chg tax expense	Thomas, Zhang 2011	Compustat	Q
23	cinvest	Corporate investment	Titman, Wei, Xie 2004	Compustat	Q
24	convind	Convertible debt indicator	Valta 2016	Compustat	Y
25	currat	Current ratio	Ou, Penman 1989	Compustat	Y
26	depr	Depreciation over PP&E	Holthausen, Larcker 1992	Compustat	Y
27	divi	Dividend initiation	Michaely, Thaler, Womack 1995	Compustat	Y
28	divo	Dividend omission	Michaely, Thaler, Womack 1995	Compustat	Y
29	dolvol	Dollar trading volume	Chordia, Subrahmanyam, Anshuman 2001	CRSP	M
30	dy	Dividend-to-price ratio	Litzenberger, Ramaswamy 1982	Compustat	Y
31	ear	Earnings announcement return	Kishore, Brandt, Santa-Clara, Venkatachalam 2008	Compustat/CRSP	Q
32	egr	Gr common shareholder equity	Richardson, Sloan, Soliman, Tuna 2005	Compustat	Y
33	ep	Earnings-to-price ratio	Basu 1977	Compustat	Y
34	gma	Gross profitability	Novy-Marx 2013	Compustat	Y
35	grCAPX	Gr capex	Anderson, Garcia-Feijoo 2006	Compustat	Y
36	grltnoa	Gr long-term net operating assets	Fairfield, Whisenant, Yohn 2003	Compustat	Y
37	herf	Industry sales concentration	Hou, Robinson 2006	Compustat	Y
38	hire	Employee gr rate	Bazdresch, Belo, Lin 2014	Compustat	Y
39	idiovol	Idiosyncratic return volatility	Ali, Hwang, Trombley 2003	CRSP	M
40	ill	Illiquidity	Amihud 2002	CRSP	M
41	indmom	Industry momentum	Moskowitz, Grinblatt 1999	CRSP	M
42	invest	Capital expenditures and inventory	Chen, Zhang 2010	Compustat	Y
43	lev	Leverage	Bhandari 1988	Compustat	Y
44	lgr	Gr long-term debt	Richardson, Sloan, Soliman, Tuna 2005	Compustat	Y
45	maxret	Maximum daily return	Bali, Cakici, Whitelaw 2011	CRSP	M
46	mom12m	12-month momentum	Jegadeesh 1990	CRSP	M
47	mom1m	1-month momentum	Jegadeesh, Titman 1993	CRSP	M
48	mom36m	36-month momentum	Jegadeesh, Titman 1993	CRSP	M
49	mom6m	6-month momentum	Jegadeesh, Titman 1993	CRSP	M
50	ms	Financial statement score	Mohanram 2005	Compustat	Q
51	mvel1	Size	Banz 1981	CRSP	M
52	mve_ia	Industry-adj size	Asness, Porter, Stevens 2000	Compustat	Y
53	nincr	Number of earnings increases	Barth, Elliott, Finn 1999	Compustat	Q
54	operprof	Operating profitability	Fama, French 2015	Compustat	Y
55	orgcap	Organizational capital	Eisfeldt, Papanikolaou 2013	Compustat	Y
56	pchcapx_ia	Industry-adj % chg capex	Abarbanell, Bushee 1998	Compustat	Y
57	pchcurrat	% chg current ratio	Ou, Penman 1989	Compustat	Y
58	pchdepr	% chg depreciation	Holthausen, Larcker 1992	Compustat	Y
59	pchgm pchsale	% chg gross margin - % chg sales	Abarbanell, Bushee 1998	Compustat	Y
60	pchquick	% chg quick ratio	Ou, Penman 1989	Compustat	Y
61	pchsale pchinvt	% chg sales - % chg inventory	Abarbanell, Bushee 1998	Compustat	Y
62	pchsale pchrect	% chg sales - % chg receivables	Abarbanell, Bushee 1998	Compustat	Y
63	pchsale pchxsga	% chg sales - % chg SG&A	Abarbanell, Bushee 1998	Compustat	Y

No.	Acronym	Firm Characteristic	Authors	Source	Freq.
64	pchsaleinv	% chg sales-to-inventory	Ou, Penman 1989	Compustat	Y
65	pctacc	Percent accruals	Hafzalla, Lundholm, Van Winkle 2011	Compustat	Y
66	pricedelay	Price delay	Hou, Moskowitz 2005	CRSP	M
67	ps	Financial statement score	Piotroski 2000	Compustat	Y
68	quick	Quick ratio	Ou, Penman 1989	Compustat	Y
69	rd	R&D increase	Eberhart, Maxwell, Siddique 2004	Compustat	Y
70	rd_mve	R&D to market capitalization	Guo, Lev, Shi 2006	Compustat	Y
71	rd_sale	R&D to sales	Guo, Lev, Shi 2006	Compustat	Y
72	realestate	Real estate holdings	Tuzel 2010	Compustat	Y
73	retvol	Return volatility	Ang, Hodrick, Xing, Zhang 2006	CRSP	M
74	roaq	Return on assets	Balakrishnan, Bartov, Faurel 2010	Compustat	Q
75	roavol	Earnings volatility	Francis, LaFond, Olsson, Schipper 2004	Compustat	Q
76	roeq	Return on equity	Hou, Xue, Zhang 2015	Compustat	Q
77	roic	Return on invested capital	Brown, Rowe 2007	Compustat	Y
78	rsup	Revenue surprise	Kama 2009	Compustat	Q
79	salecash	Sales to cash	Ou, Penman 1989	Compustat	Y
80	saleinv	Sales to inventory	Ou, Penman 1989	Compustat	Y
81	salerec	Sales to receivables	Ou, Penman 1989	Compustat	Y
82	secured	Secured debt	Valta 2016	Compustat	Y
83	securedind	Secured debt indicator	Valta 2016	Compustat	Y
84	sgr	Sales gr	Lakonishok, Shleifer, Vishny 1994	Compustat	Y
85	sin	Sin stocks	Hong, Kacperczyk 2009	Compustat	Y
86	sp	Sales to price	Barbee, Mukherji, Raines 1996	Compustat	Y
87	std_dolvol	Volatility liquidity dollar volume	Chordia, Subrahmanyam, Anshuman 2001	CRSP	M
88	std_turn	Volatility liquidity share turnover	Chordia, Subrahmanyam, Anshuman 2001	CRSP	M
89	stdacc	Accrual volatility	Bandyopadhyay, Huang, Wirjanto 2010	Compustat	Q
90	stdcf	Cash flow volatility	Huang 2009	Compustat	Q
91	tang	Debt capacity / firm tangibility	Almeida, Campello 2007	Compustat	Y
92	tb	Tax income to book income	Lev, Nissim 2004	Compustat	Y
93	turn	Share turnover	Datar, Naik, Radcliffe 1998	CRSP	M
94	zerotrade	Zero trading days	Liu 2006	CRSP	M

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